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PHENOMENONS OF PLASTIC TRANSITION IN DEFORMABLE SOLID AND TURBULENT TRANSITION IN FLUIDS

ABSTRACT

The paradigm of the possible and interpretative turbulence hearts origin and following its development in case of permanent entering into under study area fluid mediums energy with positive derivative is presented. That paradigm about the turbulent transition in fluids is described for the first timeand based on the postulate of its dominant similarity to the phenomenon of plastic deformation in solids.

KEY WORDS

Medium; deformable solid, liquid, gaseous; physical points; local thermodynamic quasi-equilibriums; plastic, turbulent transition; paradigm; postulate; dominant similarity; collapse-functions.

1. INTRODUCTION

Below we will consider the principle and qualitative description of a possible variant of spontaneous-point (in physical sense) appearance of a monofurcation (sharp violation of physical isotropy) induced in a physical point of a fluid by internal force interactions on the passage via a boundary of stability of its thermomechanical state, followed by the continuous build-up of the field development of turbulence under conditions of a dynamic process "with sharpening" (this term was originally introduced by Kapitza et al. in monograph [1] and also used in [2]). Continuous medium under consideration obey the hypothesis about local thermodynamic quasi-equilibrium into virtual physical point with measurable pressure and deterministic T_d temperature T in the presence of a stochastic noise T_{st} [3].

The proposed mechanism of local spatiotemporal fluctuation (with respect to $(\vec{x},t)$ of stressed state in a fluid) has mostly a phenomenological character, but in key respects it is based on quite convincing physical (albeit qualitative) experimental and theoretical premises.

2. BASIC PART

It is expedient to make the following preliminary remarks:

• Highly topical significance of the problem of direct description of turbulence stimulated formulation of a large number of scenarios of its "revolutionary on a microscopic scale" nucleation and subsequent evolution provided that thermomechanical energy is supplied with positive derivative to the fluid. A rather exhaustive review of physical and mathematical models developed in this field to 1968 was given in monograph [4] and a more recent period was reviewed in [5] (see also [6]). As a rule, the proposed descriptions proceed from the dynamic Navier—Stokes model for an incompressible fluid with either constant (or monotonically dependent on the temperature) viscosity coefficient (equivalent to shear rate modulus) $\mu = \dot{G}$. Unfortunately, these models (as well as the proposed neoformalism of fluid dynamics) do not disclose the internal driving force of turbulence nucleation, so that it remains a "thing-in-itself" ("Ding an sich" according to I. Kant). In this respect, it is expedient to cite J. Marsden who wrote that "For complete system of Navier—Stokes equations, we do not know any solutions which are turbulent or even that they exist" [7], since this statement is still valid. Another remarkable phrase belongs to von Kármán, who said in an opening address that "when he finally came face to face with his Creator, the first revelation he would supplicate would be unfolding of the mysteries of turbulence" (cited by [6]).

• A definite stimulus in the formation of the concept of turbulence nucleation and development presented below was given by the ideas of synergetic self-organization in open dissipative systems of various physico-bilogical and socio-anthropological origin [1, 2, 6].

• A working hypothesis adopted in what follows postulates the *dominant similarity*-conditional complementarity (key point) of the formalism of plastic (or elastic-plastic) transition in deformable solids (with Young's modulus vanishing on the flow plateau and therefore modulus B and G) and turbulent transition in fluids. Deformable solids are interpreted as classical elastic deformable mediums (up to passage through fluiditing point), that's without of the memory about theirs pre-actual configurations, as this to take place for fluid media [8, 9]. The self-consistency of the assumption concerning this similarity was evidenced, e.g., by the preceding considerations of macroscopic manifestations of dynamics in almost closely packed microstructures of deformable solids and fluid media. The marked circumstance apply to properties of the near order (for fluid-non absolute) theirs molecular and atomic structure, thermomechanics elasticity, correlations between powers of electromagnetic and kinetic fields and so on [10-12].

• In order to simplify mathematical expressions and concentrate on qualitative manifestations of the aforementioned transitions, it's enough to be limited to consideration of metomarphosis only stress tensor **P** and we will retain the most significant components in this tensor without loss of adequacy. The dynamic processes in deformable solids and fluid media will be assumed definitely isothermal (i.e., $T_d = const$).

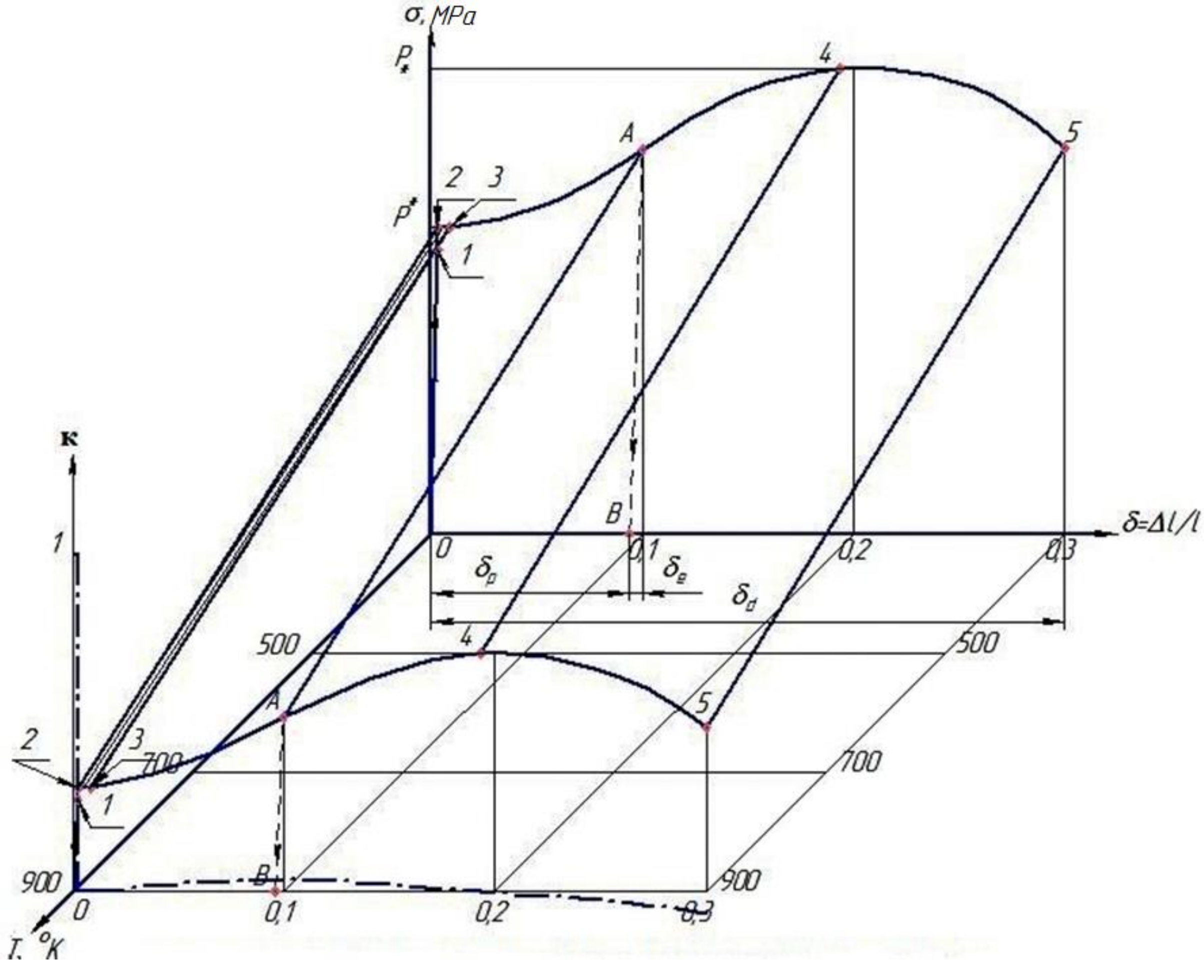

Fig. 1. Typical 3D diagram of elastic-plastic longitudinal straining of a prismatic carbon-steel sample: $\delta = \Delta l / l$ is the relative deformation; $\delta_p, \delta_e, \delta_d$ are the plastic (residual), elastic, and breaking strain, respectively; *l* is the sample length; (*1*, *4*, *5*) the lines of stress corresponsing to the proportionality limit, ultimate strength, and breaking stress, respectively; (*2*-*3*) yield plateau; **k** is the scaler collapse function**.**

Eventually, for deformable solids with allowance for the adopted assumptions, we obtain the following modified that provides a mathematical description of the process of straining with the transition vie elastic limit which is illustrated on Fig.1 (see point *2*), at least during a single action of an external tensile force:

$$\mathbf{P} = \left[- p_0 + B \int_{I_{s.1.0}}^{I_{s.1}} \boldsymbol{\kappa} dI_{s.1} \right] \mathbf{I} + 2G \int_{\mathbf{S}_{d.0}}^{\mathbf{S}_d} \mathbf{K} \cdot d\mathbf{S}_d , \qquad (1)$$

Where p_0 – initial pressure, the integrals over $\mathbf{S}_d$ (and, below, the integrals with respect to $\dot{\mathbf{S}}_d \vee \ddot{\mathbf{S}}_d$) represent a tensor form of writing integrals with respect to each current principal value of these tensors; $\boldsymbol{\kappa}$ and **K** are dimensionless scalar and ellipsoidal tensor collapse functions (working term) of $I_{s.1}$ and $\mathbf{S}_d$, respectively, and both depend on the equivalent stress P_e that usually takes place during conventional uniaxial tension of a reference sample. This parameter characterizes the properties of a material, but is independent of the type of loading. As is known, it is determined according to a selected criterion of yield at a limiting stress P^* (in Fig. 1, σ = P^*). In particular, for the Tresk—St. Venant criterion formulated for the maximum (critical) tangent stress $P_{cr} = \left\{ P_{\substack{ij \\ i \neq j}} \right\}_{\sup} = P_e = P_1 - P_3$, where P_k (k = 1, 2, 3) are the principal values of tensor **P** arranged in decreasing order [13]. Evidently, depending on the physical properties of each particular deformable solid, the plastic transition can be studied using more general criteria (e.g., the Mohr criterion), which better agree with experiment.

Below we will assume that tensor **K** is symmetric so that $\mathbf{K} \times \mathbf{I} = 0$ (vector tensor product) and its principal axes coincide with those for $\mathbf{S}_d$. This factor is, strictly speaking, an additional simplifying assumption. Note also that the integrand in the last term of Eq. (1) represents a scalar multiplication of the corresponding tensors.

In the interval of linear elastic deformation and subsequent stages of straining (up to fracture), the collapse functions (i.e., scalar $\boldsymbol{\kappa}$ and principal values K_k (k = 1, 2, 3) of tensor **K)** obey the following conditions:

$$(P_e < P^*) \to \boldsymbol{\kappa} \wedge K_k = 1 \quad \ni \quad \mathbf{K} = \mathbf{I};$$
$$(P^* < P_e^- < P_*) \to 0 \le (\boldsymbol{\kappa} \wedge K_k) < 1; \qquad (2)$$
$$(P_e^+ < P_*) \to \boldsymbol{\kappa} < 0.$$

Here, P_* is the equivalent ultimate strength; P_e^- and P_e^+ are the current equivalent stresses from left and right of the yield stress line (line *4* in Fig. 1). Relations (2) are illustrated by the plot of $\boldsymbol{\kappa}(\delta)$ (dash-dot curve) in Fig. 1. As can be seen, when the loading passes via region *1—2*, parameter $\boldsymbol{\kappa}$ of this solid material exhibits a jumplike change from 1 to 0, followed by low-sloped growth after line *3* up to the ultimate strength (line *4*), subsequent degradation of the sample structure $(d\boldsymbol{\kappa}/d\delta < 0)$, and its fracture in the vicinity of zone *5*.

Now we will pass to analysis of the turbulent transition in fluids, which will be based on the aforementioned postulate of *dominant similarity* between fluids and deformable solids in the character of tensor **P** variation when the medium attains a local critical stressed state.

Taking into account simplifying assumptions formulated above (but not assuming that stochastic component T_{st} of the temperature field is absent), we obtain the following relation for the stress tensor in a physical point of a liquid:

$$\mathbf{P} = \mathbf{P}_s + \mathbf{P}_d = -\left(p_0 + B \int\limits_{\ln \rho_0}^{\ln \rho} \boldsymbol{\kappa} d \ln \rho \right) \mathbf{I} + 2\dot{G} \int\limits_{\dot{\mathbf{S}}_{d.0}}^{\dot{\mathbf{S}}_d} \dot{\mathbf{K}} \cdot d\dot{\mathbf{S}}_d . \qquad (3)$$

The first terms in the right-hand parts of Eqs. (1) and (3) are as a matter analogous. The meaning of collapse functions $\boldsymbol{\kappa}, \mathbf{K} \wedge \dot{\mathbf{K}}$ is also much the same. Principal differences consist, first, in the fact that fluids are characterized by the prevailing effect of strain rate on the level of shear stress, which is expressed by the second term (i.e., tensor $\mathbf{P}_d$) in Eq. (3). The second difference consists in making allowance (by stepwise changes in the components of this tensor, see below) for the specific jumplike internal mesotransitions as events with enhanced macroscopic manifestation of the effects of flowability and group-average force and thermoenergetic fields in the physical point on the transition via critical values of applied tangent stresses.

Below we will consider sharp monofurcations of the principal values of tensor $\dot{\mathbf{K}}$, which should be in fact interpreted as proportional jumplike changes in the kinematic viscosity coefficient υ along the corresponding principal axes of this tensor.

It is principally possible to admit, as a simple conceptual *paradigm*, the following interpretation of a set of metamorphoses in pre-turbulent monofurcation taking place in the physical point dynamics. This paradigm involves *three decisive aspects*:

(i) Experimental fact, according to which real dropping liquids, which are always more or less multiphase and multicomponent, much weaker resist tensile stresses than compressive stresses [10].

(ii) The phenomenon of local turbulent superfluidity during totally planar supercritical tension of a physical point along two of the three principal axes of deviator $\mathbf{P}_d$ of the stress tensor.

(iii) The principle of defect revelation (recovery) of the isotropy of intrinsic physical properties of a physical point at the final stage of point monofurcation, but with lower values of the collapse function components. This principle is to a considerable degree intuitive, but it is also inspired by the Boltzmann theorem of equiprobable energy distribution over the degrees of freedom of a physical point in the statistical theory of heat capacity in classical formulation [14].

The initial stage of the turbulent transition will be considered in a local system of orthogonal $\vec{x}$, t coordinates corresponding to the principal axes of tensor $\mathbf{P}_d$ and, hence, tensors in the second term of Eq. (13). In order to simplify writing, the subscript d at the components of tensor $\mathbf{P}_d$ will be omitted. As a criterion of attaining the boundary of point monofurcation, we will use the condition of $P_{cr} = P_k\big|_{\sup} = P^*$, where k $=1 \vee 1,\ 2$ according to the adopted arrangement of principal axes of the stress tensor and its deviator $\mathbf{P}_d$. It should be noted that, in the theory of deformable solids, this criterion is used for brittle materials.

Let one of the following two situations take place in some physical point of the space-time $(\vec{x}, t^*)$ at moment $t = t^{*-}$ (i.e., on the left of t^*):

$$\left\{\left[P_1 \geq P^*, (P_2 \wedge P_3) < P^*\right] \vee \left[(P_1 \wedge P_2) \geq P^*, P_3 < P^*\right]\right\}\Big|_{t^{*-}} \quad . \tag{4}$$

Then, at $t = t^{*+}$ (i.e., on the right of t^*) the components of tensor $\dot{\mathbf{K}}$ and scalar $\boldsymbol{\kappa}$ (instead of $\dot{K}_k = 1$ (k = 1, 2, 3) and $\boldsymbol{\kappa} = 1$ at $P_k \leq P^*$) acquire in these situations the following values in accordance with the *first aspect* of the paradigm:

$$\left[(\dot{K}_1 = 0,\ \dot{K}_2 = \dot{K}_3 = 1) \vee (\dot{K}_1 = \dot{K}_2 = 0, \dot{K}_3 = 1); \boldsymbol{\kappa} = 0\right]\Big|_{t^{*+}} \tag{5}$$

Here, the second situation, whereby two of the three parameters vanish, reflects the *second aspect* of the paradigm. At the subsequent moments of time in the interval $t \in (t^{*+} + \delta t^*)$, the second term in the right-hand pert of Eq. (3) acquires the isotropic form, which corresponds to the *third aspect* of the paradigm. In this case,

$$(\dot{K}_k = \boldsymbol{\kappa} = 2/3) \vee (\dot{K}_k = \boldsymbol{\kappa} = 1/3)\ \Big|_{t^{*+} + \delta t^*}, k = 1, 2, 3. \tag{6}$$

The temporal interval of the isotropic relaxation of dynamic process in the physical point from state (5) to (6) depends on the particular microstructure. This, in particular, explains the integral form of writing of the corresponding terms in Eqs. (1) and (3), which apparently admits their description using statistical-probabilistic methods. This approach is characterized by increased complexity of results. For this reason, as the first qualitative approximation, let us estimate of the parameter under consideration based on the idea of discrete expansion of the field function in terms of their frequency-wave spectrum [15] in application to the problem of direct modeling of turbulent flows. Then for the flowing (left to right) j-th discrete component of the wave spectrum of field functions along the coordinate directions of principal values $\dot{K}_k$ of tensor $\dot{\mathbf{K}}$, the temporal interval under consideration can be evaluated as

$$\delta t_j^* \approx \left(\rho/B\right)^{1/2} æ_{s,i.j}^{-1}, \quad j = 1, 2, ..., j_{sup}\ , \quad i = inf\,, \tag{7}$$

where $(B/\rho)^{1/2}$ is the velocity of propagation of dynamic perturbations (local sound velocity) and $æ_{s,i.j}$ is the minimum wavenumber of the j-th discrete component of the wave spectrum, which probably approaches for $j \to j_{\text{sup}}$ to the Kolmogorov wave scale $æ_{s.kl}$ with negligibly small level of the energy density of turbulent pulsations [16]. The spatiotemporal transition from state (5) to (6) in the simple approximation with additional allowance for smallness of δt_j^* can be assumed to be linear.

Naturally, the jumplike and, generally speaking, stochastic (it should be recalled that $T_{st} \neq 0$) perturbation of the stressed state of a given physical point described by relations (4) – (7) is transferred to “neighboring” particles of the medium. The subsequent course of events is multivariate and cannot be described in full detail. However, this is not as important, since the practically interesting values of the field functions are determined by their averaging over an ensemble of actual realizations of the adopted theoretical model (there is a certain correlation with the ergodic Boltzmann—Gibbs hypothesis [14]). Two principally important aspects that must be necessarily satisfied by algorithms of the theoretical model are as follows:

• If, at some later moment of time $t > (t^{*+} + \delta t_j^{\ *})$ all three principal values of the stress tensor turn out to be $P_k < P^*$, then the physical point exhibits relaxation to the subcritical state with $\dot{\mathbf{K}} = \mathbf{I}$, $\boldsymbol{\kappa}$ = 1 in the reverse direction. Otherwise further decrease of the isotropy significations of the functions $\hat{\mathbf{e}} \wedge \hat{E}_{\hat{e}}$ to the schema (4), (5) are realized, namely $(\hat{\mathbf{e}} \wedge \hat{E}_{\hat{e}} = 4/9) \vee (\hat{\mathbf{e}} \wedge \hat{E}_{\hat{e}} = 1/9)$ and so on.

• In the case of continuous decrease in the level of thermomechanical energy supplied from outside to the physical point, the total number of monofurcations and their intensity decrease with recovery of the laminar flow regime ($Re < Re^*$). The opposite trend in variation of this energy leads to the establishment of a developed turbulent flow in the entire microscopic region of flow under investigation.

In concluding the consideration of one possible variant of the notion of initial stages of the turbulent transition in fluids, based on the adopted conceptual paradigm, it should be noted that subsequent investigations may lead to a conclusion (general or valid for fluids of some particular type) that the components of collapse functions exhibiting jumplike change by scheme (5) at $t = t^{*+}$ must take some values from the interval $0 \le (\dot{K}_k \wedge \boldsymbol{\kappa}) < 1, k = 1 \vee 1, 2.$ rather than vanish. Of course, based on the adopted concept, it would be also expedient to consider some other scenarios—probably more consistent with the actual picture—of the turbulent transition with sharpening of collapse functions.

For gases, in view of their typical properties of extreme flowability, unified response to compressive and tensile stresses, and expanded spectrum of thermodynamic (in particular, polytropic) processes, the criterion of turbulent transition must apparently have a more general form than that for liquids—e.g., it can be similar to the Huber—von Mises criterion used in the theory of deformable solids. These circumstances also account for the limited possibilities of simplifying the formalism used for an analysis of the phenomenon of turbulent transition in gases.

Taking into account the above considerations and retaining the most significant terms, an expression for the stress tensor of a physical point in a gas can be written as follows:

$$\mathbf{P} = -\left[p_0 + \int\limits_{\ln \rho_0}^{\ln \rho} \boldsymbol{\kappa}_f B d \ln \rho + \int\limits_{\ln T_0}^{\ln T} \boldsymbol{\kappa}_q B_\rho d \ln T \right] \mathbf{I} + 2\dot{G} \int\limits_{\dot{\mathrm{s}}_{d.0}}^{\dot{\mathrm{s}}_d} \dot{\mathbf{K}} \cdot d\dot{\mathbf{S}}_d + 2\ddot{G} \int\limits_{\ddot{\mathrm{s}}_{d.0}}^{\ddot{\mathrm{s}}_d} \ddot{\mathbf{K}} \cdot d\ddot{\mathbf{S}}_d . \qquad (8)$$

Here lover indexes *f* and *q* by **κ** means belonging to forces and heats fields accordingly, and two upper point over functions are marking their attitude to acceleration deformation (strain) processes. Expediency of the last component in the (8) right part introduction is one more insight-hypothesis. As can be seen from (8), this expression for **P** in gases is significantly more complicated than the above representations (1) and (3). However, the essence of transformations of tensor P described for the turbulent transition in fluids is also valid for gases.

In order to estimate the equivalent stress P_e for tensor $\mathbf{P}_d$ and, accordingly, express the experimentally determined critical (limiting) stress P^* of the turbulent transition in fluids, which depend on the properties of particular fluids (but neither on the type of boundaries of the region of motion studied nor on the external thermomechanical factors), it is possible to use (as it was in the study of stress—strain states of deformable solids) theoretical and experimental data on the values of these parameters for flows in the canonical domains. These may include flows over flat surface, Couette flow between parallel plates, or flow in round cylindrical tubes.

3. CONCLUSION

Fully recognizing complexity of the object of this investigation and the topical significance of problems considered above, the author would be grateful to all members of scientific community engaged in the dynamics of strongly inhomogeneous continuous media for any constructive discussion of the results of this study.

NOMENCLATURE

B	Isothermal bulk modulus;
B_ρ	Isochoric bulk modulus;
$G \wedge \dot{G} \wedge \ddot{G}$	Isothermal shear modulus, its variation rate and acceleration;
$\mathbf{I}$	Unit tensor;
$I_{s.1}$	First invariants of tensors **S**;
$\mathbf{K},\dot{\mathbf{K}},\ddot{\mathbf{K}}$	Tensor collapse functions;
$K_k, \dot{K}_k$	Principal values of the tensosrs $\mathbf{K},\dot{\mathbf{K}}, k=1,2,3$;
$\boldsymbol{\kappa}$, $\boldsymbol{\kappa}_f$, $\boldsymbol{\kappa}_q$	Scalar collapse functions;
$\mathbf{P},\mathbf{P}_s,\mathbf{P}_d$	Stress tensor, spherical stress tensor, and stress deviator;
P_{ij}, P_e, P^*, P_*	Components of tensor $\mathbf{P}$ $(i \wedge j = 1,2,3)$; equivalent stress, elastic limit, and ultimate strength;
P_k	Principal values of the tensor $\mathbf{P}_d$, k=1,2,3;
Re, Re^*	Reynolds number, critical Reynolds number;
$\mathbf{S}$	Strain (deformation) tensor;
$\mathbf{S}_d \wedge \dot{\mathbf{S}}_d \wedge \ddot{\mathbf{S}}_d$	Deviators of the strain tensor, strain rate and acceleration tensors;

T, T_d, T_{st} Absolute temperature, absolute deterministic and stochastic temperatures;

$t, \vec{x}, (\vec{x}, t)$ Time, radius vector, and space-time, respectively;

æ$_s$ Spectral wavenumber;

$\mu = \dot{G}$ Dynamic viscosity coefficient (equivalent to shear rate modulus);

ρ Density of medium;

ω_s Spectral frequency;

$\wedge, \vee, \ni$ Logical symbols for "and, or, therefore."

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