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DISCRETE SPECTRUM DECOMPOSITION OF FIELD FUNCTIONS FOR STRONGLY INDIGNANT DYNAMICS OF CONTINUOUS MEDIUMS

ABSTRACT

To raising of the calculating procedures stability on direct modelling of continuous media motions in dynamical processes «with sharpening» the method of field functions specific decomposition to frequency-wave numerical intervals is propounded and uncovered as a matter. It's revealed, that under satisfaction of a «filter property», which usually physical inherent to this function, present decomposition procedure allows, in principle, truncating to the right along decreasing rates of a boundary conditions for partial field functions on discrete with the greatest frequency-wave numbers to such for initial discrete..

KEY WORDS

Continuous; media; turbulence; direct modelling; frequency-wave spectrals, numbers; field functions, partial; discrete decomposition; influence rate; cascading transfer; turbulent pulsations energy.

1. INTRODUCTION

The method described below was briefly outlined in proceedings [1] of a local-scale conference that could make it hardly accessible for scientific community. This consideration has led to a more detailed and clearly determined disclosure of the substance of the procedure for expansion of field functions by their frequency-wave spectrum (*FWS)*, as applied to the tasks of simulation of considerably heterogeneous continuum dynamics.

Describing continuum motion in conditions of intense and largely chaotic pulsations of field substances all across the Kolmogorov spectrum of frequency ω_s and wave $æ_s$ (vector) ∍ (ω_s, $æ_s$) numbers *FWN*, i.e., with their "aggravated" dynamics, generates problems of raising the stability of numerical implementations and establishing the average (or pseudo-average) values determining these variations of variables, which are of practical interest. The above issue is faced, e.g., at attempts of direct computer-based reproduction of turbulent and particularly intermittent laminar-turbulent streams. Such media are also known to be characterized by uncertainty in ranges of averaging field functions by time, and in the general case, by space, as well [2], at deduction of Reynolds-averaged Navier-Stokes equations [3].

Hereinafter, we use the logical symbols $\ni, \in, \wedge, \vee, \|..\|, \cup, \rightarrow, \uparrow$ - "so that", "element of", "and", "or", "norm", "union", "it follows", "increase", as well as $[0]^m$ – a value of the *m*-th order of smallness.

2. BASIC PART

The approach suggested below should be regarded as a method of possible overcoming or, at least, attenuation of the aforesaid difficulties.

Let the dynamics of a continuum in a finite four-dimensional $3D_t$ area V with boundary ∂V $(\bar{V} = V \cup \partial V)$ be described with a closed system of equations

$$\frac{d\,\mathbf{f}_k}{dt} = \mathcal{F}_k\left\{\overline{\mathbf{f}_1, \mathbf{f}_n}; \vec{F}_k, q_k\right\}, k = \overline{1,n} \tag{1}$$

with partial derivatives in the right side, i.e. for arguments t, x_r, where x_r (r =1,2,3) are projections of radius vector $\vec{x}$ in the Cartesian reference system, generally to the second order, by space. In system (1) d/dt is a substantional operator of differentiation by time, $\mathbf{f}_k$ are tensor, vector or scalar field functions, $\mathcal{F}_k$ are operators of, generally speaking, nonlinear transformations of functions in braces; $\vec{F}_k$ are external three-dimensional and surface force fields, and q_k are energy (mostly, thermal) fields of effects on physical point, which may depend on values of $\mathbf{f}_k$ at ∂V, where the over-bar means enumeration. Here and below, a physical point is an ultimately small (in virtual macroscopic representation) particle of a continuous medium, which content to the hypothesis about "local thermodynamics quasi-equilibrium" [4].

Write the boundary conditions in the following general form

$$\overset{\iota\,\circ}{\mathbf{f}_k} = \overset{\iota}{\mathcal{F}}{}_k^{\circ}(\overline{\mathbf{f}_1, \mathbf{f}_n}), (\bar{x}, t) \in \partial V, k = \overline{1,n}; \quad \overset{\iota\,\circ}{\mathbf{f}_k} = \begin{cases} \mathbf{f}_k^{\circ}, \iota = \varnothing - \iota \text{ is empty set;} \\ \partial \mathbf{f}_k^{\circ} / \partial x_r, \quad \iota = \bullet, r = 1,2,3. \end{cases} \tag{2}$$

The composition and form of conditions of (2) ensure *existence*, but (as distinct from Hadamard boundary problems set in a classically correct way [5]), *not necessarily uniqueness* of an analytical solution, let alone a numerical solution. This, apart from effects of rounding errors, is associated with a component of functions $\overset{\iota}{\mathbf{f}_k}, \overset{\iota\,\circ}{\mathbf{f}_k}$ commonly observed and assumed to be random. Presumably, this component increasingly manifests itself along with approximation of the computational mesh cells to the scale of physical point (superhigh numbers (ω_s, **æ**$_s$)) and causes certain aberration of conditions (2).

Manifestations of stochastic behavior of dynamic variables and taking into account possible background noise in boundary conditions confirm an imperative on the result of establishing average values of field functions by an *ensemble* of computer implementations of motion under examination of “aggravated” continua. In a generalized sense, here, as mentioned above, we can see an abstract correlation with the Boltzmann-Gibbs ergodic hypothesis [6].

Next, assuming that the continuum under consideration satisfies the *filter condition*, i.e., permanent decline of pulsation amplitude along with a growth of frequency $\omega_s\uparrow$ and wave $\text{æ}_s\uparrow$ numbers, so that $(\omega_s, \text{æ}_s)\uparrow$, present every function $\mathbf{f}_k$ as a sum of its partial summands $\mathbf{f}_{kj}$ determined at $j\uparrow$ on an increasingly fine computational mesh, namely

$$\mathbf{f}_k = \sum_{j=1}^{m} \mathbf{f}_{kj} + \varepsilon_k, \qquad \|\varepsilon_k\| \in \left[0, c_k\,[0]^m\right] \tag{3}$$

where $\mathbf{f}_{kj}$ is the k-th field function determined at the j-th increment of *FWS*, $\|\varepsilon_k\|$ is conventional rate of deviation of ε_k of the sum specified in the right side of equation (3) from $\mathbf{f}_k$, and c_k are finite positive numbers. Enumeration of increments is exercised in an ascending order, e.g., by an order, of *FWN*. Metric limits of cell $3D_t$ of computational mesh are determined uniquely with intervals $[(\omega_s, \text{æ}_s)_{j.\inf}, (\omega_s, \text{æ}_s)_{j.\sup}]$ of the relevant j-th increment of *FWS*. Obviously, $(\omega_s, \text{æ}_s)_{j-1.\sup} = (\omega_s, \text{æ}_s)_{j.\inf}$.

It is clear that the first increment, index $j = 1$, comprises metric parameters of an entire closed computational domain of continuum motion $\bar{V} = V \cup \partial V$ (which is also true for other increments at $j>1$), but with the range of *the smallest FWN*. On convergence, with an acceptable dispersion of computational process, there may be established distributions of $\mathbf{f}_{k,j=1}$, which may be regarded as pseudo-average values of $\mathbf{f}_k$ and practical applications of the theory that are (first and foremost) of interest. The last expansion increment ($j = m$) is approximation to the scale of the left boundary of applicability of the local thermodynamic quasi-equilibrium hypothesis. Intervals, in particular, the first one (j=1), as well as the total number of increments m, depend on physical properties of the continuum under examination, conditions of its motion, proximity of physical point to solid parts of the boundary of ∂V (if any), the degree of decline of pulsation amplitudes with $j\rightarrow m$, as well as resources of computer equipment in use.

In line with (3), match each equation of system (1) against an *iterative* (e.g., according to Gauss-Seidel method) totality of equations

$$\frac{d\,\mathbf{f}_{kj}^{(n)}}{dt} = \mathcal{F}_k^{*}\left\{\left(\sum_{i=1}^{m} \mathbf{f}_{1i}^{*(n)}{}_{i\le j} \wedge \mathbf{f}_{1i}^{*(n-1)}{}_{i>j}\right), \overline{\left(\sum_{i=1}^{m} \mathbf{f}_{ni}^{*(n)}{}_{i\le j} \wedge \mathbf{f}_{ni}^{*(n-1)}{}_{i>j}\right)}; \vec{F}_{kj}^{\,*}, q_{kj}^{*}\right\},\ k=\overline{1,n};\ j=\overline{1,m};\ \mathbf{f}_{kj}^{*} \equiv \mathbf{f}_{kj} \tag{4}$$

with boundary conditions

$$\partial \mathbf{f}_{kj}^{\circ} = \mathbf{f}_{kj}^{\circ} - \mathbf{f}_{k,j-1}^{\circ *},\ \mathbf{f}_{k,0}^{\circ} \equiv 0,\ \mathbf{f}_{km}^{\circ} = \mathbf{f}_{k}^{\circ} + \varepsilon_{km}, \qquad \left\|\varepsilon_{km}\right\| = \mathbf{c}_{km}\,[0]^m, \tag{5}$$

additively meeting conditions of (2) in a descending order, with *m* growing to a *prognostically* small and therefore allowing truncation "from the right" of the value of rate of influence $\left\|\overset{\iota}{\varepsilon}_{km}\right\|$ produced by solution of the boundary problem with boundary condition $\partial\overset{\iota\,0}{\mathbf{f}}_{km}$ on functions $\mathbf{f}_{k,j=1}$.

In relations (4), (5): (*n*) is the number of the current iteration, $\overset{\iota}{c}_{km}$ are some nonnegative finite numbers, superscript * denoting mapping of each function $\mathbf{f}_{ki} \vee \overset{\iota\,0}{\mathbf{f}}_{k,j-1}$ and operators $\mathcal{F}_k, \overset{\iota}{\mathcal{F}}{}^0_k$ on the *j*-th increment of FWS. Such transformation, e.g. for functions $\mathbf{f}_{ki}$ is expressed as follows

$$\underset{i\neq j}{\mathbf{f}^*_{ki}} = \int\limits_{\delta V_j} W_j(\vec{\xi},\tau)\mathbf{f}_{ki}(\vec{x}-\vec{\xi},t-\tau)dV, \int\limits_{\delta V_j} W_j(\vec{\xi},\tau)dV = 1,$$
$$\delta V_j = \delta V_{x.j}\delta\tau_j, \delta V_{x.j} = \prod_{r=1}^{3}\delta x_r, \tag{6}$$

where δV_j is four-dimensional volume of the computational mesh cell uniquely associated with the *j*-th increment of *FWS* [(ω$_s$,**æ**$_s$)$_{j.\inf}$, (ω$_s$,**æ**$_s$)$_{j.\sup}$]⟶ δV_j, W_j is suitable weight function (m^{3}sec)$^{-1}$.

It would appear reasonable that the right part of equations (4), as distinct from initial equations (1), additionally comprises summands $-\left\{\frac{d^*}{dt}\left(\sum_{i=1, i\neq j}^{m} \underset{i<j}{\mathbf{f}_{ki}^{*(n)}} \wedge \underset{i>j}{\mathbf{f}_{ki}^{*(n-1)}}\right)\right\}$.

Note that mapping of $\underset{i<j}{\mathbf{f}_{ki}}$ on the *j*-th 3D$_t$ computational mesh corrects functions $\mathbf{f}_{kj}$ in a relatively weak way. On the contrary, with the mesh in question superimposed with functions $\underset{i>j}{\mathbf{f}_{ki}}$, $\mathbf{f}_{kj}$ would be smoothed considerably to an extent increasing with *i*↑. This is what, in essence, makes the factor of raising the stability of computational algorithms at computer-based simulation of strongly disturbed continuum dynamics, and makes possible, in principle, to establish the values of field functions $\overline{f_k} \approx f_{k,j=1}$ that would be average for $(t,\vec{x}) \in \overline{V}$.

It is also noteworthy that the above mentioned procedure, as applied to fluid media, actually reproduces a known property of cascaded transfer of energy fluctuations of turbulent fields from their macroscales to miniscales [7,8].

3. CONCLUSION

A crucial and apparently difficult matter remains open: which could be correlation between results of numerical implementation of the problem of describing strongly heterogeneous continuum dynamics complicated with presence, in the general case, of a low-level random field influence, according to algorithm of [3 – 6] and immediately based on initial statement (1, 2)? The above mentioned requires further thorough examination of proper mathematic aspects of the problem in question within a more general problem of simulation of bifurcating and developed supercritical motions of continua.

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