Morgunov G.M.
Dr. Sci. Professor
National Research University “Moscow Power Engineering Institute” Moscow city
ggm@mpei.ru

RATIONAL PHENOMENOLOGICAL MODELS OF EXTERNAL FIELDS EFFECT IN CONDITIONS OF NON-EQUILIBRIUM THERMOMECHANICS OF CONTINUOUS MEDIA

ABSTRACT

The paper presents naturally determined simulation neomodels of dependencies of a group of physical coefficients in structural-parametric expressions for temperature and stress tensor, as well as in partial laws and regularities describing per totality external mechanical and thermal spatial-temporal effects on selected particles of continuous medium - definitely solid, liquid or gaseous - on fundamental matter-field substances: quantity of motion, density, increment of internal energy.

It is shown that, if assumptions adopted in the paper meet the physical nonrelativistic bases of phenomena under consideration, then the models presented transform these factors established or subject to be determined experimentally in conditions, which are generally of global equilibrium, or those of stationary one-dimensional thermal mechanical tests, into conditions of the hypothesis of local thermodynamic equilibrium assumed in this study.

A strong form is obtained for Clapeyron equation for non-equilibrium macrothermodynamic processes.

KEY WORDS

Continuous media, non-equilibrium thermodynamics, simulation, functions, temperature, Joule law, inversion, compression pressure, shearing pressure, physical coefficients, experimental, current, fundamental substances, density, internal energy increment, quantity of motion, equivalent stress.

INTRODUCTION

This article should be considered as further detailed elaboration, and in some aspects, also refinement of physical bases and neoformalism in simulation of dynamics of strongly disturbed continual media. The integral substance of the study undertaken is presented conceptually and in a primary form in publication [1] and further in articles [2 – 4]. These papers are assumed to be known including attitudes and designations adopted therein.

The field functions – *compression* pressure (see below) p and temperature T (assumed to be deterministically measurable, at least for fluid media), as well as coefficients (parameters, moduli) such as those of volume elasticity/shear, thermal conductivity, etc. are featured in physical ideas of factors of $3D_t$ *effect* on each element of continuous medium [4] and constitute, with a completeness difficult for assessment, a projection of selected particles of disturbed states of the microcosm contained therein onto macroscales.

Identifiers under consideration, in their turn, make part of members of *action* of fundamental equations of the state of medium [4]. These members are considered in the Eulerian representation of motion, yet in a way mutually related, by means of the above equations, with the Lagrangian representation of *rates of variation* of the matter-field substances to be determined: density ρ, internal energy increment $\Delta\varepsilon$ and quantity of motion related to unit of mass, i.e., velocity $\vec{v}$. Here, the position of physical points of moving medium at any moment is assumed to match the fixed Eulerian spatial coordinates $x_k (k = 1,2,3)$.

Physical coefficients to be analyzed below contain information, in a form that is unfortunately reduced (for known reasons), on results of *group* interactions of molecular-atomic structures of the medium. Owing to virtually unlimited plurality and an objectively existing deficit in knowledge of specifics of macro-manifestations of the above interactions, as well as technical difficulties in holding accurate field tests in their general $3D_t$ setup, these physical coefficients are generally established, or may be verified, in conditions of a thermomechanical process, which is *globally* equilibrium, or stationary single-dimensional one, for moduli of *rates* of volume and shear strain*.

We shall call the aforesaid circumstances «*e*» - *conditions* (from Engl. "*Equilibrium"*). At the same time, it has to be noted that the slash symbol («/») between words in the text means tautological *"or"*.

In actual situations of definitely heterogeneous motions of media considered here, and accordingly, of substantially non-equilibrium "in-large" thermodynamics, the empirical coefficients obtained by the above method will most likely deviate from their current values. This implies an imperative of acceptably determined correction, a kind of extrapolation "inwards" of coefficients under consideration on conditions of existence of just local thermodynamic equilibrium assumed in this study.

Besides, in terms of physical essence the solutions of a system of fundamental equations of dynamics of continuous media (see, e.g., [1]), if these solutions exist for the algorithmized kind of such equations, are, as was

---

* Theoretically with arguments: radius of spherical coordinates of a spherical sample of medium in the former case, and a coordinate in the direction of shear rate, in the latter.

mentioned, substances $\rho(\vec{x},t), \Delta\varepsilon(\vec{x},t), \vec{v}(\vec{x},t)$. Then, it is clear that the above functions *T* and *p,* along with physical coefficients being carriers of experimental information for «*e*»-*conditions*, must, along with the necessity of extending this information "inwards" be expressed in a particular way through current values of the aforesaid substances.

Proceeding now to the main part of the article, which provides a description of methods for solving the problems set, we shall state preliminary notes on assumed simplifications of formalism, which have virtually no principal relevance here, yet considerably facilitate analysis.

● When describing thermal processes, their stochastic component is left beyond consideration, i.e., we will assume *T* as a determined component of temperature. Therefore, we have omitted the prefix "quasi" before the word equilibrium in the relevant *local* hypothesis.

●● Supercritical conditions of dynamics are not considered, as these are beyond the scope of this study. As was specified, there are no *collapse-functions* in tensor expressions [1].

●●● The insight hypothesis presented in [1] on *possible* effect of *acceleration* of strains $\ddot{\mathbf{S}}$ on tensor thermomechanical phenomena has required further elaboration of formalism. Therefore, neither the aforesaid hypothesis, nor its consequences are considered here.

Finally, note that the following logical designations are used in the text and in formulae: $\wedge, \vee, \leftrightarrow, \ni, \in, \cup, \cap, \rightarrow$, Ø, $\uparrow$, which have the following meaning, in order of their enumeration: *and, or, one-to-one correspondence, so that, belongs to, union, intersection, it follows, empty set, ascendancy.*

MAIN PART

The study outlined in the introduction will be performed according to a specific procedure. For a selected particle of the medium, there will be established dependencies of the following factors on $\rho, \Delta\varepsilon, \vec{v}$:

● Function of *T* and physical coefficients in *partial* laws (Joule law, Fourier law) and in other thermal effects of thermodynamics (rate of thermal strains, supply of radiant energy).

●● Function of *p* in spherical component $\mathbf{P}_s$ of stress tensor $\mathbf{P}$ and moduli of elastic compression/volume elasticity, i.e., strain of *volume* of the medium particle from directly mechanical or initially thermal external effects, and the rate of variation of this strain, included in an expression for the same. Further on, for the function of *p,* for a circumstance explained below, instead of the generally accepted name *pressure* (absolute), we will use the term *pressure* (absolute) of (volume) *compression*, with adjectives in brackets omitted.

●●● Moduli of shear/strain *of the shape* of volume of medium particle included in deviator $\mathbf{P}_d$ of tensor $\mathbf{P}$ and the rate of variation of this shear, also occurring for reasons stated in the previous paragraph.

● Now then, express temperature through density and internal energy increment. To this effect, apply Joule law, which is related to the first law of thermodynamics and follows from the definition of specific heat capacity $c_{\mathbf{v}}$ for an isochoric process. With thermodynamic transformations tentatively considered as equilibrium ones, we have

$$d\Delta\varepsilon\big|_e = \partial Q_{\mathrm{v}}\big|_e \,, \tag{1}$$

$$\partial Q_{\mathrm{v}}\big|_e = c_{\mathrm{v,e}}(\bar{\mathrm{v}},T)\partial T, \quad (c_{\mathrm{v,e}} \wedge c_{\mathrm{v}}) > 0, \quad \bar{\mathrm{v}} = \mathrm{v}/\mathrm{v}_0 \,. \tag{1a}$$

Here, Q_{v} is the function of *process* (the symbol «∂» in the abbreviation of intermediate differential) in the form of an amount of heat delivered to the medium particle for the “frozen” volume (specific) $\bar{\mathrm{v}}$. The subscript *e* - general (under a vertical line) and in the designation of $c_{\mathbf{v}}$, as well as in subsequent expressions for experimental parameters, will denote the results of its/their experimental measurements obtained, for the circumstances stated in the introduction, in *«e»-conditions*. Further, $\Delta\varepsilon$ is an increment of *total* internal energy. Here, we keep in mind that this function of *state* of medium, and in the case of enthalpy and entropy, as well, is determined for macro-scale phenomena, with accuracy to an additional constant, only. However, as based on the last inequality in (1a), we conclude that $\Delta\varepsilon$, as an integral for *T*, is an increasing function of temperature that assumes no negative values: $T\uparrow \rightarrow \Delta\varepsilon\uparrow$, which means that it would be appropriate to suppose that

$$\Delta\varepsilon = \varepsilon\big|_{T>T_{\mathrm{inf}}} - \varepsilon\big|_{T_{\mathrm{inf}}} = \int_{T_{\mathrm{inf}}}^{T} c_{\mathrm{v}} dT \ni \Delta\varepsilon > 0, \tag{2}$$

where T_{inf} is temperature lower than any current temperature from the range $T \in [T_i, T_{\mathrm{sup}}]$ for any specific thermal process, $T_i = T_{\mathrm{inf}} + \delta T, \delta T > 0$ is arbitrarily small positive value.

Now that we have established relation (2), we agree, for the sake of shortening the notation, to omit the symbol Δ and the word “increment”.

Having, in (1), (2) and hereafter, the involved substance ε, as such, rather than its thermal component ε_q alone, is related to consideration of a known experimental fact: even in case of a truly adiabatic and mechanically purely shear motion of an actual continuous medium, this dynamic process, which is undoubtedly non-linear in terms of energy involved, is not isoentropic/non-equilibrium and, hence, to an extent, irreversible. As a matter of fact, when considering the area of spatial-temporal distribution of mechanical/force ε_f and thermal ε_q components of ε in the set-theoretical aspect, it is possible to imagine in an abstract way (as applied to the specific thermomechanical situation considered in this paragraph) as the following union of its components $\varepsilon = \varepsilon_f \cup (\varepsilon_f \cap \varepsilon_q) \cup \varepsilon_q$, which is also clarified in fig. 1.

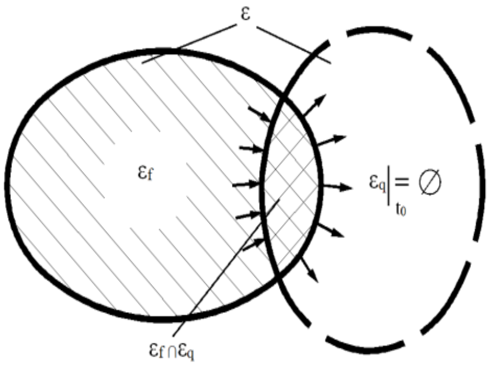

Fig. 1. Illustration of directivity of mechanism of internal dissipative dispersion of energy with generation of a part of component ε_f of thermal form of energy ε_q for an adiabatic dynamic process: t_0 is precondition of absence of increment ε_q.

The area of intersection of sets ε_f, ε_q and the directivity of energy transitions according to fig. 1 corresponds to causal and relaxation-temporal transformation of a part of ε_f into ε_q. In other words, what is stated is an effect of dissipation by some members of component ε_f of this mechanical component of internal energy, including generation of heat *"out of itself"* distributed on the scales of the medium macroparticles, with the micro-scale nature of its origination. The formalism developed in this theory reflects, as a complement to the aforesaid, existence of this crucial manifestation of the second law of thermodynamics:
- note the section of analysis of fundamental equation of energy balance in the paper-precursor of [1], which establishes availability in the component ε_f of summands of dissipative action of shear stresses;
- note interrelations $\partial T \leftrightarrow d\varepsilon \ni \partial T = 0 \leftrightarrow d\varepsilon = 0,$ being in line with the aforesaid and following from relations (1), (1a);
- occurrence of partial transformation of ε_f into ε_q will also be seen in an expanded model presentation obtained below (18) for tensor $\mathbf{P}_d$ and follow from the content of the paragraph completing the main part of the paper.

Now, re-write integral (2) for *«e»-conditions* with consideration of a convention of shortened notation assumed for this expression

$$\varepsilon\Big|_e = \int_{T_i}^{T} c_{\text{v}.e}(\bar{\text{v}}, T)\partial T, T \in [T_i, T_{\sup}], \quad (3)$$

where specific volume $\bar{\text{v}}$ acts as a parameter similar, e.g., to a composite variable, or its imaginary part for areas of presentation in direct Laplace and

Fourier transformations of operator calculus. Hence, integration in (3) is exercised along isolines $\overline{\mathrm{v}}$, which is also suggested by designation ∂ of intermediate differential *T*. Thus a one-to-one relation $\varepsilon \leftrightarrow T$ is established for the specified conditions of implementation of the relevant experiment.

Heat capacity c_{v} beyond the zone of phase transitions must, according to its physical essence, constitute an unambiguous, positively defined, limited and continuous, along with its first derivatives, function of $\overline{\mathrm{v}}$ and *T*, admitting experimental definition, at least, in «*e*»*-conditions.* Hence, what takes place is coefficient $\beta_{\mathrm{v}.e}=c_{\mathrm{v}.e}^{-1}$, which is reverse and unambiguously correspondent thereto. We will assign this coefficient with a *tentative* term *back parameter* (from Engl. «*back*»). Relations (1, 1a), with obvious equality $\overline{\mathrm{v}}=(\rho/\rho_0)^{-1}$, allow for the back parameter to undergo the following transformation by *arguments* (name of independent variables justified in *«e»-conditions*) $\beta_{\mathrm{v}.e}(\overline{\mathrm{v}},T) \leftrightarrow \beta_{\rho.e}(\rho,\varepsilon)$.

Then, we assume that for *all* the physical coefficients featured in the subsequent calculations, which have been experimentally established in *«e»-conditions* by this moment, or are subject to experimental determination according to an algorithm being elaborated here and generally considered as functions of $(\overline{\mathrm{v}} \vee p(\overline{\mathrm{v}},T)) \wedge T$, it could be possible, *if necessary,* to exercise the following unambiguously correspondent transitions by arguments

$$(\overline{\mathrm{v}} \vee p,T)\big|_e \leftrightarrow (\rho,T)\big|_e \leftrightarrow (\rho,\varepsilon)\big|_e . \tag{4}$$

Such transition is always possible, if on 2*D* surfaces of relations of coefficients under consideration on *initial* arguments, say, $c_{\mathrm{v}.e} = c_{\mathrm{v}}(\overline{\mathrm{v}},T)\big|_e$, it is possible to distinguish a countable number of intervals, within which derivatives for these arguments would not change their sign. This condition is hereinafter considered satisfied. Then, given the above reasoning, we will exercise *inversion* of relations (1, 1a), by expressing temperature in the overall $3D_t$ motion of the medium with the following integral over *ε,* with density fixed at each moment (note that there is no subscript *e* here!)

$$T = T_0 + \int_{\varepsilon_0}^{\varepsilon} \beta_\rho(\rho,\varepsilon)\,d\varepsilon, \qquad T_0 \geq T_i , \tag{5}$$

Where *ρ*, similarly to $\overline{\mathrm{v}}$ in integral (3), is considered as a parameter, while «*e*»*-conditions* act as *reference/*primary data.

The sequence of transitions resulting in establishing relation $T = T(\rho,\varepsilon)$ is qualitatively shown in fig. 2*a* – 2*d*.

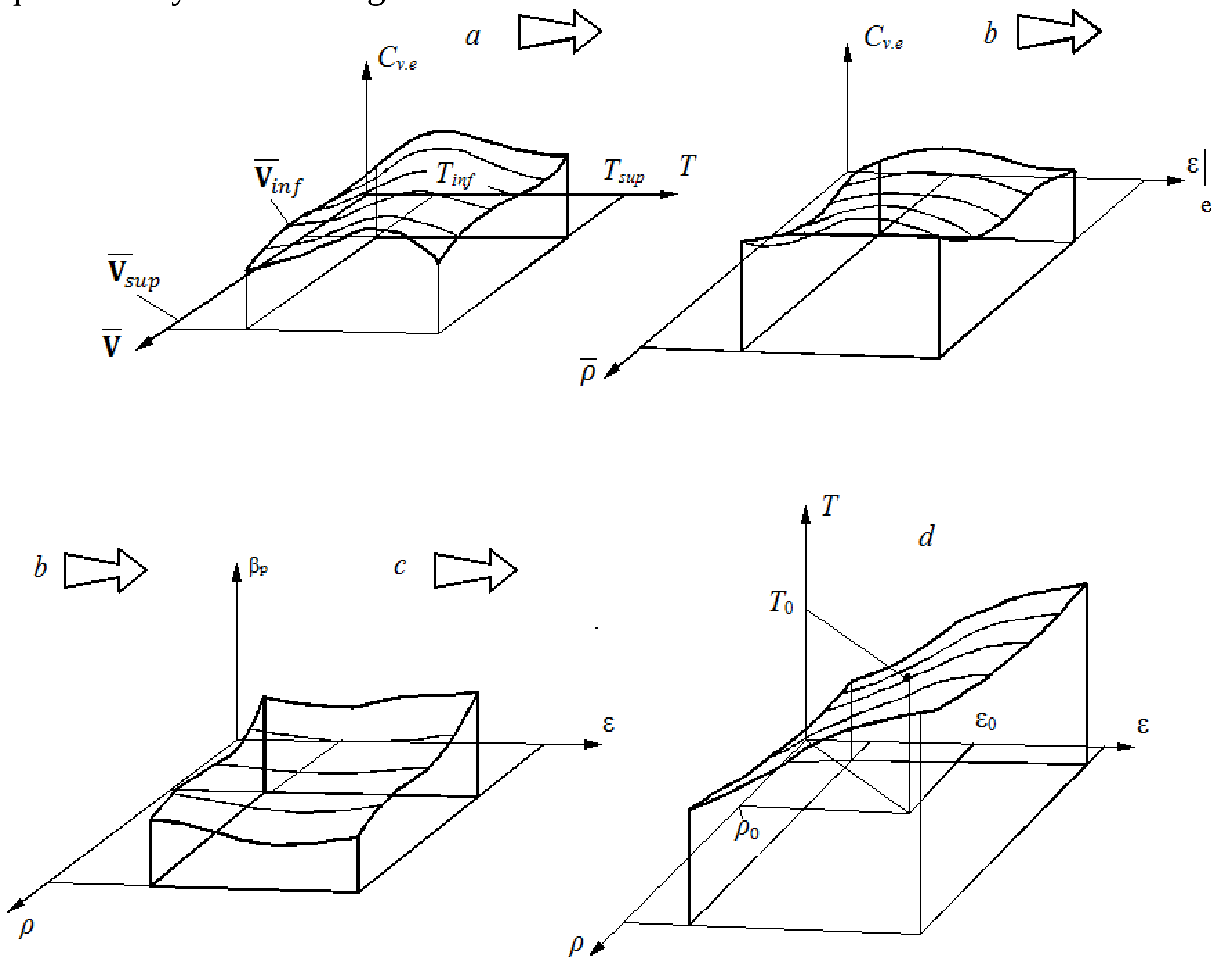

Fig. 2. Illustration of transition by arguments for heat capacity $c_{v.e}$:*a*, *b*; establishing back parameter $\beta_\rho(\rho,\varepsilon)$: *c*; obtaining relation $T = T(\rho,\varepsilon)$: *d*.

For identification of back parameter β_ρ, present its total differential as

$$d\beta_\rho = \frac{\partial \beta_\rho}{\partial \rho} d\rho + \frac{\partial \beta_\rho}{\partial \varepsilon} d\varepsilon = (1-\mu_\beta)\left(\frac{\partial \beta_\rho}{\partial \rho}\bigg|_e d\rho + \frac{\partial \beta_\rho}{\partial \varepsilon}\bigg|_e d\varepsilon \right). \quad (6)$$

Here, the function of $(1-\mu_\beta)$ is a dimensionless integrating factor, component $\mu_\beta(\rho,\varepsilon)$ is discrepancy. Members with subscript *e* under a vertical line are considered known throughout the physically possible range of variation of substances ρ and ε. Discrepancy μ_β must meet the non-homogeneous differential equation (what is essential is the sequence of differentiation)

$$\frac{\partial}{\partial \rho}\left(\frac{\partial \beta_\rho}{\partial \varepsilon}\bigg|_e \mu_\beta \right) - \frac{\partial}{\partial \varepsilon}\left(\frac{\partial \beta_\rho}{\partial \rho}\bigg|_e \mu_\beta \right) = \frac{\partial}{\partial \rho}\left(\frac{\partial \beta_\rho}{\partial \varepsilon}\bigg|_e \right) - \frac{\partial}{\partial \varepsilon}\left(\frac{\partial \beta_\rho}{\partial \rho}\bigg|_e \right). \quad (7)$$

With the right part of (7) converted into zero (as it is also the case in equations (13), (19) below), the integrating factor is assumed to be equal to one, so that μ_β = 0. Integrating (6) according to the rules of calculation of contour integrals, we have (as a particular method of actions)

$$\beta_\rho = \beta_{\rho.0} + \int_{\rho_0,\varepsilon_0}^{\rho,\varepsilon_0} \frac{\partial\beta_\rho}{\partial\rho}\bigg|_e (1-\mu_\beta)d\rho + \int_{\rho,\varepsilon_0}^{\rho,\varepsilon} \frac{\partial\beta_\rho}{\partial\varepsilon}\bigg|_e (1-\mu_\beta)d\varepsilon\,,$$

or, when presented in a general form

$$\beta_\rho = \beta_{\rho.0} + \int_{\rho_0,\varepsilon_0}^{\rho,\varepsilon} \left[\frac{\partial\beta_\rho}{\partial\rho}\bigg|_e (1-\mu_\beta)d\rho + \frac{\partial\beta_\rho}{\partial\varepsilon}\bigg|_e (1-\mu_\beta)d\varepsilon \right] ,\rho_0 \wedge \varepsilon_0 \to t = t_0\,. \tag{8}$$

At special case μ_β=1 equation (7) is contented as identity and in equality (8) under integral expression for given time moment t is conversioned into zero. In equalities (5) – (8) for differentials ρ and ε related to $3D_t$ motions of medium, we have

$$d(\rho \vee \varepsilon) = \partial_{\vec{x}}(\rho \vee \varepsilon) \wedge \partial_t(\rho \vee \varepsilon), \qquad \partial_t = (\partial u)(\vec{\mathbf{1}}\cdot\vec{\nabla}), \quad \partial u = |\partial\vec{u}|\,, \tag{9}$$

where $\partial_{\vec{x}}$ and ∂_t are, respectively, linear and non-linear operators of intermediate differentials for t with fixed $\vec{x}$ and in the direction of velocity $\vec{v}$ with unit vector $\vec{\mathbf{1}} = \vec{v}/v = d\vec{u}/du$ with fixed t. It goes without saying that the differential of vector of displacement (including deformations) $d\vec{u}$ of physical point is collinear to vector of velocity $\vec{v}$.

Differentials (9) may be expressed through right sides of fundamental equations, presented, among others, in [1] under numbers (1), (3):

$$d\rho = \frac{d\rho}{dt}dt = \rho\, dI_{s.1} = \rho \overset{\bullet}{I}_{s.1}\, dt\,; d\varepsilon = \frac{d\varepsilon}{dt}dt = \rho^{-1}\mathbf{E}dt, \;\; \mathbf{E} = \mathrm{P}\cdot\cdot\overset{\bullet}{\mathbf{S}} + \rho q, \tag{10}$$

where $I_{s.1}$, $\overset{\bullet}{I}_{s.1}$ are the first invariants of the strain and strain rate tensors $\mathbf{S}$, $\overset{\bullet}{\mathbf{S}}$; q is the specific amount of heat delivered to the particle of the medium from outside in a unit of time.

We will consider the method described in algorithm (5) – (10) for expanding experimental information on properties of continuous media in their thermodynamically non-equilibrium "in-large" states to be also fulfilled for other experimental coefficients and parameters in particular manifestations of factors of *effect*. According to the range of questions considered in [1], such parameters include coefficients of:

Thermal conductivity λ in Fourier law of conductive heat transfer, rate of thermal strains $\mathcal{G}$, conductive share of radiant energy absorbed by the particle θ and other experimental physical parameters, which it will be necessary to introduce, in particular, at detailed elaboration of formalism of description of dynamic processes, with consideration of *memory* of pre-actual states of solid deformable bodies, or regularities of development of wall turbulence for fluid media.

●● We now proceed to the question of rational simulation of compression pressure p in tensor $\mathbf{P}_s$ using the results presented in the previous paragraph. As in [1] (but with consideration of equation (5) and assumptions simplifying analysis outlined in the introduction above), we consider function *p* dependent on ρ, ε and on their first derivatives with respect to *t*. Then, generalizing relevant expressions in [1] and using relations (9), (10), for differential *p* we will successively write:

$$dp = dp_1 + dp_2;\quad (\rho \wedge \varepsilon) > 0;$$

$$dp_1 = \frac{\partial p}{\partial \ln\rho} d\ln\rho + \frac{\partial p}{\partial \ln\varepsilon} d\ln\varepsilon,\quad dp_2 = \frac{\partial p}{\partial \frac{d}{dt}\ln\rho} d\frac{d}{dt}\ln\rho + \frac{\partial p}{\partial \frac{d}{dt}\ln\varepsilon} d\frac{d}{dt}\ln\varepsilon;$$

$$d\ln\rho = dI_{s.1},\quad d\ln\varepsilon = (\rho\varepsilon)^{-1}\mathbf{E}\,dt;\quad d\frac{d}{dt}\ln\rho = d\overset{\bullet}{I}_{s.1},\quad d\frac{d}{dt}\ln\varepsilon = d\left((\rho\varepsilon)^{-1}\mathbf{E}\right).$$

We introduce for $3D_t$ motion of medium “current” (with angle brackets to be hereinafter omitted) dynamic moduli of compression and rate of its variation, for ε and ρ fixed alternately:

$$B_\varepsilon = \frac{\partial p}{\partial \ln\rho},\quad \overset{\bullet}{B}_\varepsilon = \partial p \Big/ \partial\frac{d}{dt}\ln\rho;\quad B_\rho = \frac{\partial p}{\partial \ln\varepsilon},\quad \overset{\bullet}{B}_\rho = \partial p \Big/ \partial\frac{d}{dt}\ln\varepsilon. \tag{11}$$

Hence, the following equalities apparently follow

$$dp_1 = B_\varepsilon d\ln\rho + B_\rho d\ln\varepsilon,\quad dp_2 = \overset{\bullet}{B}_\varepsilon\, d\frac{d}{dt}\ln\rho + \overset{\bullet}{B}_\rho\, d\frac{d}{dt}\ln\varepsilon.$$

Proceeding from current models to experimental ones for *“e” - conditions*, with their correction through introduction of integrating factors (similar to notation of equation (7)), for total differential of *p* we will have:

$$dp = (1-\overset{\iota}{\mu}_B)\left(\overset{\iota}{B}_{\varepsilon.e}\, dL\overset{\iota}{n}\rho + \overset{\iota}{B}_{\rho.e}\, dL\overset{\iota}{n}\varepsilon\right), \iota = \varnothing, \bullet \text{ (to be summed up), -} \tag{12}$$

with two additional equations for discrepancy $\overset{\iota}{\mu}_B$

$$\frac{\partial}{\partial L\overset{\iota}{n}\rho}\left(\overset{\iota}{B}_{\rho.e}\,\overset{\iota}{\mu}_B\right) - \frac{\partial}{\partial L\overset{\iota}{n}\varepsilon}\left(\overset{\iota}{B}_{\varepsilon.e}\,\overset{\iota}{\mu}_B\right) = \frac{\partial}{\partial L\overset{\iota}{n}\rho}\overset{\iota}{B}_{\rho.e} - \frac{\partial}{\partial L\overset{\iota}{n}\varepsilon}\overset{\iota}{B}_{\varepsilon.e}, \iota = \varnothing, \bullet \text{(not to be summed)} \tag{13}$$

In (12), (13), as in [1], we use operator $L\overset{\iota}{n}$: $Ln = \ln \wedge \overset{\bullet}{Ln} = \frac{d}{dt}\ln$.

Integration (12) results in the following expression for the spherical part of stress tensor

$$\mathbf{P}_s = p\mathbf{I} = \left\{ p_0 + \int\limits_{L\overset{\iota}{n}_0\rho,\, L\overset{\iota}{n}_0\varepsilon}^{L\overset{\iota}{n}\rho,\, L\overset{\iota}{n}\varepsilon} \left[\left(1-\overset{\iota}{\mu}_B\right)\overset{\iota}{B}_{\varepsilon.e}\, dL\overset{\iota}{n}\rho + \left(1-\overset{\iota}{\mu}_B\right)\overset{\iota}{B}_{\rho.e}\, dL\overset{\iota}{n}\varepsilon\right]\right\}\mathbf{I}. \tag{14}$$

Here **I** is unit tensor, summing over $\iota = \varnothing, \bullet$.

If we refer to expressions (9) obtained in [1] for *p* as the *weak* form of generalization of Clapeyron equation for non-equilibrium macrothermodynamical processes, then it would be natural to define equality (14), along with equations (13), as the *strong* form of such generalization.

●●● In the concluding section of the main part of the paper, we will now present expansion of the formalism of theory [1], which is related to more rational simulation of deviator components $\overset{\iota}{\mathbf{P}}_d$ of stress tensor $\mathbf{P}$.

In the previous paragraph, function p of forceful action retaining or changing *volume* $\delta\mathbf{v}$ of the medium particle, was identified as *compression pressure*. Expedience of introduction of such term for p follows from the below circumstance.

In dynamics, the medium particle, apart from the aforesaid factor, is also subjected (which is obvious) to forceful action of variation of the *shape* of its surface $\delta\boldsymbol{s}$, and hence, shear strains and rates of these strains.

Introduce <u>new</u> *scalar positive* functions of field

$$\overset{\iota}{\tilde p} = \frac{1}{2}\sum_{k=1}^{3}\left|\overset{\iota}{P}_{d.k}\right|, \quad \overset{\iota}{\tilde s} = \frac{1}{2}\sum_{k=1}^{3}\left|\overset{\iota}{S}_{d.k}\right|, \quad \iota = \varnothing, \bullet, \tag{15}$$

where $\overset{\iota}{P}_{d.k} \wedge \overset{\iota}{S}_{d.k}$ are the principal values of respective tensors $\overset{\iota}{\mathbf{P}}_d \wedge \overset{\iota}{\mathbf{S}}_d$ with properties of *orthogonality* of $\delta\boldsymbol{s}$ *and equality to zero* of their sums.

Note that for symmetrical deviator tensors, there always exist limited principal values (including zero), as the discriminant of the relevant cubic characteristic equation $D \le 0$. What is said is demonstrated in Fig. 3 for algebraically ranked principal values, e.g., of tensor $\mathbf{P}_d$: $P_{d.1} \ge P_{d.2} \ge P_{d.3}$. In Fig. 3, reduced parameters are used $\lambda_3 = P_{d.3}/P_{d.1}$, $\lambda_2 = P_{d.2}/P_{d.1} \ni \lambda_1 = 1$, $P_{d.1} > 0$.

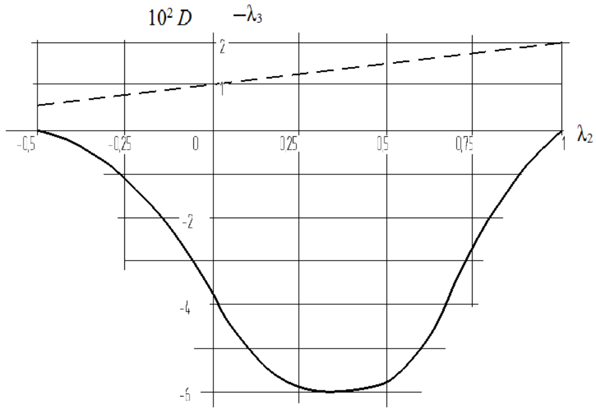

Fig. 3. Dependencies of discriminant *D* of characteristic equation for principal values of symmetrical deviator tensor (solid line) and λ_3 (dashed line) on λ_2.

Now, we state that the procedure of actions for determination of principal directions and principal values of symmetrical tensors under consideration is well-known and introduce shear moduli:

$$\overset{\iota}{G} = \left(\frac{\partial \overset{\iota}{\tilde p}}{\partial \overset{\iota}{\tilde s}}\right) > 0, \ \iota = \varnothing, \bullet. \tag{16}$$

The structure of expressions (15), (16) enables to regard functions $\overset{\iota}{\tilde p}$ and $\overset{\iota}{\tilde s}$ as virtual ones, i.e. corresponding at each moment of time to principal directions of tensors determining these variables, *shear pressure and parameter of anisomorphism/*strain $\delta\mathbf{s}$ and the same for the rates of shear/variation of this shape.

It is to be noted that establishing $\overset{\iota}{\tilde{p}}$ according to the first equality in (15) meets the value of the largest tangential strain

$$\overset{\iota}{P}_{\tau.\sup} = \frac{1}{2}\left(\overset{\iota}{P}_{d.1} - \overset{\iota}{P}_{d.3}\right) \tag{17}$$

for purely shearing uniaxial tension of a cylinder-shaped specimen of a hard structural material (ι = Ø), as well as gradient-free one-dimensional Couette flow along a perfectly smooth flat wall (ι = •). Here, it is easy to make sure of total complementarity of determination of modulus $\overset{\bullet}{G}$ under (16) Newton law of viscous friction.

Also note that parameter $\overset{\iota}{P}_{\tau.\sup}$ in (17) is used, or may be used at fixation of achievement of critical states of continuous media, e.g., Tresk-St.Venant criterion for solid bodies.

A similar criterion for fluid media, alternative to the option considered in [1] $\overset{\bullet}{P}_e = \overset{\bullet}{P}_1\Big|_{\sup} = \overset{\bullet}{P}*$ (in [1], the upper points are omitted) may be presented as follows

$$\overset{\bullet}{P}_{\tau.cr} = \overset{\bullet}{P}_e, \quad \overset{\bullet}{P}_e = \overset{\bullet}{\tilde{p}} = \overset{\bullet}{P}*, \quad \overset{\bullet}{P}* = \frac{1}{2}\left(\overset{\bullet}{P}_{d.1} - \overset{\bullet}{P}_{d.3}\right)_{\sup}, \tag{17a}$$

where $\overset{\bullet}{P}_e$ is limiting equivalent stress related to tensor $\overset{\bullet}{\mathbf{P}}_d$, $\overset{\bullet}{P}*$ is limiting value of tangential stress for the Couette flow mentioned in the previous paragraph.

Here, it should be emphasized that the postulate *of dominant similarity* of phenomena of plastic nature in solid bodies and those of turbulent nature in fluid media of transitions put forward in [1] was related, for the latter, by default, to flows beyond the zones of the appreciable action of wall regularities, i.e., to phenomena of development of *free* turbulence (also see [1], clarification on page 24 “We will leave beyond consideration the questions…”).

Further procedures leading to dependencies of moduli $\overset{\iota}{G}$ on ρ and ε in many respects are similar, in terms of their sequence and formal presentation, to the algorithm of (6) – (10). Therefore, with intermediate calculations left out, we will now present the final results:

$$\mathbf{P}_d = 2\overset{\iota}{G}\overset{\iota}{\mathbf{S}}_d = 2\left\{\overset{\iota}{G}_0 + \int_{\rho_{0.i},\varepsilon_{0.i}}^{\rho_i,\varepsilon_i}\left[\left(1-\mu_{G.i}\right)\frac{\partial \overset{\iota}{G}}{\partial \rho_i}\bigg|_e d\rho_i + \left(1-\mu_{G.i}\right)\frac{\partial \overset{\iota}{G}}{\partial \varepsilon_i}\bigg|_e d\varepsilon_i\right]\right\}\overset{\iota}{\mathbf{S}}_d \tag{18}$$

(to be summed over $\iota \wedge i$) with four additional equations for each discrepancy $\mu_{G.i}$

$$\frac{\partial}{\partial \rho_i}\left(\left.\frac{\partial \overset{\iota}{G}}{\partial \varepsilon_i}\right|_e \mu_{G.i}\right)-\frac{\partial}{\partial \varepsilon_i}\left(\left.\frac{\partial \overset{\iota}{G}}{\partial \rho_i}\right|_e \mu_{G.i}\right)=\frac{\partial}{\partial \rho_i}\left(\left.\frac{\partial \overset{\iota}{G}}{\partial \varepsilon_i}\right|_e\right)-\frac{\partial}{\partial \varepsilon_i}\left(\left.\frac{\partial \overset{\iota}{G}}{\partial \rho_i}\right|_e\right), \qquad (19)$$

not to be summed over $\iota \wedge i$.

Meaning of the lower index i attached to the arguments ρ,ε: i = 1 – absence of any operations on the same, and with i = 2 – action of operator d/dt. For fluid media, expressions (18), (19) should virtually be used with $\iota = \bullet$, and for solid bodies, mostly, with upper index $\iota = \varnothing$.

Also note the following: as, based on the form of the right side of equality (18) for $\mathbf{P}_d$, the equation of balance of internal energy ε must comprise summand $\overset{\bullet}{G}\overset{\bullet}{\mathbf{S}}_d^2$ (also see expression (10) in [1]), then, even with a purely adiabatic *dynamic* process, and in absence at the initial time point of gradients ε, availability of the aforesaid summand will lead to generation of a positive (see (15), (16)) increment of ε, and hence, that of temperature T. This circumstance is a manifestation of the effect of increasing entropy with partial transition of mechanical/force share ε_f of internal energy into its thermal form ε_q "out of itself" and serves as an additional factor of legitimacy of having equation (5) involved for determination of T as function of ρ (here, parameter), as well as increment of *total* internal energy ε.

From the principal point of view, dependency of physical coefficients considered in the paper on substances: ρ, ε and rates of their variation, - is an undisputable empirical fact. With this a posteriori assumption adopted, we establish that equations (7), (13), (19) for respective residual errors do extend information on properties of continuous media obtained in *«e»-conditions* of their examination over conditions of non-equilibrium thermal mechanics.

CONCLUSION

Undoubtedly, the extraordinary complexity of the formalism presented is a factor assumed involuntarily that is softened however by the permanent progress observed in development of capabilities of computing tools.

REFERENCES

[1] Morgunov G. M. A Neoformalism approach to the dynamics of continuous media / World intellectual property organization. Publ. date 05.02.2015, 44p. [In Russian]
https:// patentscope.wipo.int/search/en/detail.jsf?docId=WO2015016736
[2] Morgunov G. M. Phenomenons of plastic transition in deformable solid and turbulent transition in fluids / Materials of the V international scientific and practical extra-mural conference «Fundamental science and technology-

promising developments V», vol. 1, 24-25 Feb. 2015. North Charleston, USA, pp. 147-156. – ISBN:978-1508657552. http://dev.science-publish.ru
[3] Morgunov G. M. Discrete spectrum decomposition of field functions for strongly indignany dynamics of continuous mediums / Materials of the V international scientific and practical extra-mural conference «Fundamental and applied sciences today V», vol. 1, 30-31 march. 2015. North Charleston, USA, pp. 144-148. – ISBN:978-15181565684. http://science-publish.ru/node/2.
[4] Morgunov G. M. Truth episteme in continuous mediums dynamics / Materials of the VI international scientific and practical extra-mural conference "Topical areas of fundamental and applied research VI", vol. 1, 22-23 June. 2015. North Charleston, USA, pp. 144-148. – ISBN:978-15147117370. http://science-publish.ru/node/2.