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DEDUCTION OF ACTION FACTORS IN NEOFORMALISM OF DYNAMICS OF CONTINUOUS MEDIA

ABSTRACT

Based on the conventional idea on kinetics of continuous media (J.-L. Lagrange – L. Euler) and deductive analysis, there have been obtained expressions of components of strain tensors, velocities and accelerations of strains through constituents of the speed vector and their partial time derivatives.

KEY WORDS

Continuous media, dynamics, actions, rates of change, Lagrangian, Eulerian, representations of motion, kinetic tensors.

INTRODUCTION

In the paper, there are discussed and formalized two essential aspects of a paradigm (being developed successively and interactionally) of phenomenological $3D_t$ description of dynamics of strongly disturbed continuous media being in different, yet stable proper aggregate states [1 – 5], namely:

- deduction is exercised of causal factors of *force action* on a selected particle of the media, those factors being formalized in the reference system and hence causing origination and further transformation, in the *Lagrangian* representation of motion, *of rates of change* of the fundamental substances of its thermodynamic state, more specifically, density ρ, internal energy ε and specific quantity of motion / momentum, i.e., velocity $\vec{v}$ [4, 5];

- a conclusion is given on dependencies of tensors $\overset{\iota}{\mathbf{S}}$ ($\iota = \varnothing$, □,□□; $\varnothing$ is empty set) making part of the aforesaid effect factors, on components of substance $\vec{v}$ and their partial time derivatives *t*. Here (also see [1]), $\mathbf{S}, \dot{\mathbf{S}}$ and $\ddot{\mathbf{S}}$ are strain tensors, velocity and acceleration of variation of volume and shape of elementary particles of continuous medium, respectively.

The undertaken resorting to clarification of the above aspect is determined by extraordinary complexity of the subject under study and an imperative of particularization of the method of its physical-mathematical simulation.

MAIN PART

The initial thesis of the aspect under consideration is the following judgment on mathematical description of the cause and effect relations of the *"action-rate-rate of change"* in fundamental dynamic equations of conservation/balance of substances ρ, ε, $\vec{v}$ [4].

The functions of *rates of change* of the above entities (generally recorded in the left sides of the stipulated equations) are considered in the *Lagrangian* representation of motion of continuous media, i.e., in the coordinate system (hereinafter referred to as the Cartesian coordinate system) $\vec{x} \vee x_k (k = \overline{1,3})$ "tracking" *the trajectory* of displacement in time *t* of each its elementary particle (through to the extremely small macro-scales of the physical point) with an *only* absolutely independent argument *t*. We shall refer to this "tracking" $3D_t$ coordinate system as an *"L-system"*.

The factors of *action* (and accordingly, for definiteness, the right sides of the said balance equations) are set on a dense spatial set of points of the rigid *Eulerian* 3D coordinate system $\vec{\mathbf{x}} \vee \mathbf{x}_k (k = \overline{1,3})$ as a function of *four* independent arguments $(\mathbf{x}_k, t)$. We will assign this reference system with a term *"E-system"*.

According to the well-known Helmholtz theorem, motion of particles of a medium may be represented as superposition $\vec{x} = \vec{x}_\bullet + \vec{u}$ made up of translational displacement with focal point/center $\vec{x}_\bullet$ of mass δ*m* of each of them as *an almost perfectly solid* body, as well as asymmetrical and symmetrical, relative to the point in question, motions $\vec{u}$ of turning/ rotation and deformational variations of the volume and shape of these particles, respectively.

Choose in the 3D space of the *"E-system"* a fixed reference point $\vec{\mathbf{x}}$ and observe passing through it with time of centers $\vec{x}_\bullet$ of masses δ*m* of particles of the medium, assuming that their motion is set in an *"L-system",* and masses δ*m* are the same and constant. It follows that there is time-independent equality $\vec{x}_\bullet = \vec{\mathbf{x}}$. Then, considering, at the specified conditions and on scales of the particles being tracked, variations of their volume and shape with time, for partial derivatives $\frac{\partial x_i}{\partial \mathbf{x}_j} (i \wedge j = \overline{1,3})$ making part of components of tensor $\mathbf{S}$, we establish that

$$\frac{\partial_j x_i}{\partial \mathbf{x}_j} = \frac{\partial_j u_i}{\partial \mathbf{x}_j}, \tag{1}$$

as, by condition, point $\vec{x}_\bullet$ is frozen and, hence, $\partial_j x_{\bullet i} = 0_\bullet$. Now, as translational motion of small physical entities of medium allows, as was mentioned above, their being regarded as motion of an almost perfectly body, derivatives with respect to $\mathbf{x}_j$ of convective transfer of field substances (including, as is clear, transfer of an amount quantity of motion/momentum with the medium particles passing through point $\vec{\mathbf{x}}$) are also equal to *physical zero* [4]. Here, we mention in passing that subscripts j (and further t, as well), with intermediate differentials in the numerators (in this particular case, see (1)) emphasize their appurtenance to the respective arguments.

Now, taking into account transient variations of vector $\vec{u}$ and legitimacy of transmutations of operators of differentiation with respect to t and spatial coordinates $\mathbf{x}_k$, by virtue of independence of these arguments in the reference *"E-system",* for the relevant increments of the right side of equality (1), we successively find

$$\partial_t \frac{\partial_j x_i}{\partial \mathbf{x}_j} = \partial_t \frac{\partial_j u_i}{\partial \mathbf{x}_j} = \frac{\partial_t}{\partial t}\frac{\partial_j u_i}{\partial \mathbf{x}_j} dt = \frac{\partial_j}{\partial \mathbf{x}_j}\frac{\partial_t u_i}{\partial t} dt = \frac{\partial_j}{\partial \mathbf{x}_j}\delta v_i dt \ , \tag{2}$$

where δv_i is transient increment of the i-th component of velocity $\vec{v}$, for differential of which, in its turn, the following operations are true

$$\partial_t \frac{\partial_j \delta v_i}{\partial \mathbf{x}_j} = \frac{\partial_j}{\partial \mathbf{x}_j}\frac{\partial_t \delta v_i}{\partial t} dt = \frac{\partial_j}{\partial \mathbf{x}_j} a_{l.i} dt \ , \tag{3}$$

where $a_{l.i}$ is the i-th component of *local* acceleration $\vec{a}_l$ of increment of velocity $\vec{v}$ of the medium particles passing through point $\vec{\mathbf{x}}$ of the *"E-system"*. Now, suppressing superfluous indices in designation of differentials in numerators and integrating (3), and then, twice, (2) by time, we obtain

$$\frac{\partial v_i}{\partial \mathbf{x}_j} = \frac{\partial}{\partial \mathbf{x}_j}\left(v_{0.i} + \delta v_i \Big|_t^0\right) = \frac{\partial}{\partial \mathbf{x}_j}\left(v_{0.i} + \int_0^t a_{l.i} d\tau\right), v_{0.i} = v_{0.i}(\vec{\mathbf{x}},0); \tag{4}$$

$$\frac{\partial u_i}{\partial \mathbf{x}_j} = \frac{\partial}{\partial \mathbf{x}_j}\left(u_{0.i} + \delta u_i \Big|_0^t\right) = \frac{\partial}{\partial \mathbf{x}_j}\left(u_{0.i} + \int_0^t \Big(\int_0^{\leftarrow t} a_{l.i} d\tau\Big) d\tau\right), u_{0.i} = u_{0.i}(\vec{\mathbf{x}},0). \tag{5}$$

Here, subscript 0 means the initial value of the relevant function. The head arrow above the inner integral in expression (5) indicates that the increment of outer integral is determined directly by the increment of inner integral at each current quanta dt.

In case the dynamic process reaches a *practically* settled regime, beginning, e.g., from time moment t_+ to a specified accuracy, and according to (4), we have

$$\vec{a}_l \approx 0, t{>}t_+, \qquad \vec{v}(\vec{x}, t{>}t_+) \approx \vec{v}(\vec{x}) \approx \vec{v}_0 + \delta\vec{v}\Big|_0^{t_+} \quad \ni \quad \int_0^{t>t_+} a_{l.i} d\tau \approx \int_0^{t_+} a_{l.i} d\tau \tag{6}$$

Besides, by analogy with (6), it is not difficult to note the relevant relations for vector of strains $\vec{u} = \vec{u}_0 + \delta\vec{u}\Big|_0^{t>t_+}$, as well. For the initially settled motion of continuous medium, it is obvious that $\vec{v}(\vec{x},t) = \vec{v}_0(\vec{x})$, and in conditions of a global equilibrium thermodynamic process, $\vec{v} = 0, \vec{u}(\vec{x}) = \vec{u}_0(\vec{x})$. Also note that the components of differentials $d\vec{x}(t)$ of the *"L-system"* and $d\vec{\mathbf{x}}$ of the *"E-system"* are related as $dx_k = dx_{\bullet k} + du_k = \left(1 + \frac{\partial u_k}{\partial \mathbf{x}_k}\right) d\mathbf{x}_k$.

Here, it is to be pointed out that the structure of expressions (4), (5), which comprise, in a direct and defining way, the function of field $\vec{a}_l$, are positively, though indirectly, correlated with the above-presented (see [1]) hypothesis on dependence of stress tensor **P** (in a confined volume of a selected medium particle) not only on tensors $\mathbf{S} \wedge \dot{\mathbf{S}}$, but also on strain acceleration tensor $\ddot{\mathbf{S}}$.

To conclude, we present, using the previous dependences, expanded expressions for components of the said kinematic tensors

$$\mathbf{S} = \frac{1}{2}\left(\frac{\partial\left[u_{0.i} + \Upsilon_u\left(a_{l,i}\right)\right]}{\partial \mathbf{x}_j} + \frac{\partial\left[u_{0.j} + \Upsilon_u\left(a_{l,j}\right)\right]}{\partial \mathbf{x}_i} \right), \quad \Upsilon_u = \int_0^t \Big(\int_0^{\leftarrow t} \_\_ d\tau \Big)\, d\tau \;;$$

$$\dot{\mathbf{S}} = \frac{1}{2}\left(\frac{\partial\left[v_{0.i} + \Upsilon_v\left(a_{l,i}\right)\right]}{\partial \mathbf{x}_j} + \frac{\partial\left[v_{0.j} + \Upsilon_v\left(a_{l,j}\right)\right]}{\partial \mathbf{x}_i} \right), \quad \Upsilon_v = \int_0^t \_\_ d\tau ; \qquad (7)$$

$$\ddot{\mathrm{S}} = \frac{1}{2}\left(\frac{\partial a_{l,i}}{\partial \mathbf{x}_j} + \frac{\partial a_{l,j}}{\partial \mathbf{x}_i} \right), i \wedge j = \overline{1,3}.$$

Obviously, the first two lines in representations (7) may be presented without particularization and formally in a generally accepted form, with using the extreme left derivatives with respect to $\mathbf{x}_{j \vee i}$ in equalities (4), (5). What also follows from the form of presentation of summands in (7) is an essential result, which lies in that the components of tensors $\overset{\iota}{\mathbf{S}}$ ($\iota = \varnothing, \bullet, \bullet\bullet$) only comprise constituents of velocity $\vec{v}$ and their partial derivatives with respect to *t*, i.e., these tensors contain no "superfluous" variables.

Here, it is also appropriate to make the following judgment. The variant of origination of turbulent monofurcation in fluid media, which is based on the postulate of *"dominant similarity"* put forward in this theory (see [1, 2]), included dependence of the relevant critical parameter on tensor $\dot{\mathrm{S}}$, only. However, at present, there are no grounds, apart from speculative conjectures, to exclude possible effect produced on this parameter by constituents of the force functions of *action* with tensors S and $\ddot{\mathrm{S}}$.

CONCLUSION

Based on ideas on registration of *rates of change* of fundamental substances of continuous media in the Lagrangian "tracking" reference system (factor of *consequence*), and that of functions of *action* – in the Eulerian "rigid" coordinate space (factor of *cause*), there have been obtained linear integro-differential expressions for strain tensors, their velocities and accelerations without "superfluous" variables. Components of the aforesaid tensors only depend on constituents of the velocity vector and their partial time derivatives.

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