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PARADIGM OF THE AMPLIFICATION IN KINETICS OF CONTINUOUS MEDIUMS AND GENERATED BY IT’S CONSEQUENCES

ABSTRACT

On the basis of promoted paradigm about evolution of processes of the elementary macroparticles turning/torsion, as with proper deformation, by general rules of the compressable, deformable and heat-conducting continuums the imperative at the passing to formalism of *quasi-symmetrical* thermomechanics of such mediums has been established.

For this purpose the needed modification of the previously suggested phenomenological model of the *symmetrical* thermomechanics of the present mediums has been carried out.

KEY WORDS

Continuous medium, paradigm, amplification, quasi-symmetrical, thermomechanics, formalism, modification, substantial derivative

INTRODUCTION

By decisive factor of this extended in comparison with [1] conception in following rotary-shearing model of the continuous medium kinematics (fig.1).

The movement $\vec{x}$ of any *marked* point ***d*** of also *marked* elementary medium particle with velocity $\vec{v} = d\vec{x}/dt$, as well as acceleration $\vec{a} = d\vec{v}/dt = \vec{a}_l + \vec{a}_{cv}$, where $\vec{a}_l \wedge \vec{a}_{cv}$, local and convective accelerations, is fixed by *Lagrangian reference system*, i.e. *L-system* $(t, \vec{x}(t))$ $(t \wedge \vec{x}$ – in thin type) and considers in the form of two motions superposition $\vec{x} = \vec{x}_c + \vec{u}$. This sum is added from forward motion $\vec{x}_c$ with velocity $\vec{v} = d\vec{x}/dt$ as well as acceleration $\vec{a}_c = d\vec{v}_c/dt$ of the conditional, but fixed focal point ***c***, for example, center of mass, into the given *marked* particle and from additional the change of point ***d*** position $\vec{u} = \vec{u}_0 + \delta\,\vec{u}$ relatively of the focus ***c***. Here symbol δ stands for a moving increment at time of function $\vec{u}$ in *L-system*; index 0 is indicator of its initial meaning. Motion $\vec{u}$ occur with velocity $\vec{v}_d = \frac{d\vec{u}}{dt}$ and acceleration $\vec{a}_d = \frac{d\vec{v}_d}{dt}$ of a *full* termomechanics deformation/distortion of *marked* particle.

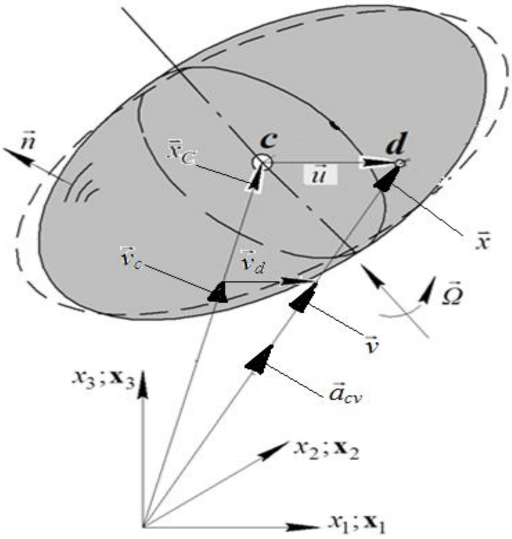

Fig. 1. Scheme of the kinematic displacement (darkened contour) and distortions of shear (dotted line), as well as torsion (dot-dash axis and vector $\vec{\Omega}$) of the marked particle of medium.

On the other hand, for functions of the *actio*n [2] and accordingly in *Eulerian reference system, i.e. E-system* $(\boldsymbol{t},\vec{\mathbf{x}})$ ($\boldsymbol{t} \wedge \mathbf{x}$ – in bold type), mentioned above thermomechanics distortions, at every instant time moment and to within terms of a higher order infinitesimal, can be represented in terms, which incorporates in the additional form items of the *pure* deformation of the *marked* particle volume and form, as well as its no rigid turn relative of the virtual axis with focus ***c***. Indeed,

$$\delta\,\vec{u} = (\mathbf{S}+\mathbf{A})\cdot\delta\,\vec{\mathbf{x}},\; 2\mathbf{S} = \left(\vec{\nabla}_{\vec{\mathbf{x}}}\vec{u}\right)^* + \vec{\nabla}_{\vec{\mathbf{x}}}\vec{u}\,,\; 2\mathbf{A} = \left(\vec{\nabla}_{\vec{\mathbf{x}}}\vec{u}\right)^* - \vec{\nabla}_{\vec{\mathbf{x}}}\vec{u}\,, \qquad \text{(I)}$$

where $\mathbf{S} \wedge \mathbf{A}$ is symmetric and anti-symmetric tensors (second tensor – with all principal meanings are equal to zero in the isotropy way) of *proper* deformation of the *marked* particle and its *non-rigid* turning accordingly. Increment δu is expressed in terms of known differential diades with independent arguments $\boldsymbol{t} \wedge \mathbf{x}_k, k=\overline{1,3}$ (see further formulas (6)). In relation (I) also $\vec{\nabla}_{\vec{\mathbf{x}}}$ is Hamelton's operator in *E-system*, and upper asterisk is symbol of transposition.

Result (I) have for the following analysis principal valve, which be defined of the qualitative dissimilarity from a steady state representation about turnings/rotates of the particles of medium (usually – fluid) as taking place by regularities of perfectly (more often use prefix «quasi») a solid. Essentially, such interpretation contains in Helmholtz theorem about expansion motion of particles at continual mediums. For the reasons given above word-combination «According to the well-known Helmholtz theorem…» in fourth indention of main part of the paper [1] must be omit.

We believe that the stated definition matched to physical presentations about real properties motion being investigated here mediums. Present point of view is, in particular, unconditional on torsions of the real rigid bodies. At present for fluid mediums it is no discovered reasons to ignore permissible in principle existence and definite contribution to corresponding factors of the *action* additional and introduced further phenomenologicaly added by volume *outward* forces and moment of *inner* forces couple into, fundamental equations of conservation with regard generally happening to rotary as well as and particularly-turbulent motions.

MAIN PART

Formulated in introduction conceptual tenet have as an outcome imperative of the modification of primarily accepted phenomenological model of continuous mediums dynamics [3]. Present modification is based on next premises.

● *Principle of disappearance divergence* of «action-rate of change» relations [2], which realizes in representations of *E⋀L-systems* indication [1], is satisfied. Here term «*action*» we mean at an extended interpretation in comparison with similar term in Hamelton's principle.

●● Additional external forces and moments actions on choses medium particles are referred to category of mass/volume disturbances as a consequence from theory of solid torsion and accepted further ways of these disturbances formalization.

●●● Phenomenological model description indicated disturbances includes components of anti-symmetric tensors $\overset{\iota}{\mathbf{A}} = \left(\overset{\iota}{\mathrm{A}}_{ij}\right) = -\left(\overset{\iota}{\mathrm{A}}_{ji}\right), i \wedge j = \overline{1,3}$. Here for *additional* variable $\overset{\iota}{\vec{w}}$ have: $\vec{w} = \vec{u}, \iota = \varnothing$; $\dot{\vec{w}} = \vec{\boldsymbol{v}}_d = \partial\vec{u}/\partial \boldsymbol{t},\ \iota = \bullet$; $\ddot{\vec{w}} = \vec{\boldsymbol{a}}_d = \partial\vec{\boldsymbol{v}}_d/\partial \boldsymbol{t},\ \iota = \bullet\bullet$. In this connection for components of the *full* deformation acceleration $\vec{\boldsymbol{a}}_d$ are correct equalities (see [1]) $\partial \boldsymbol{a}_{d.i}/\partial \mathbf{x}_j = \partial \boldsymbol{a}_{p.i}/\partial \mathbf{x}_j$, where $\boldsymbol{a}_{p.i}$- projections of partial derivative $\partial v_i/\partial \boldsymbol{t}$ from velocity $\vec{v}$ at $\boldsymbol{t}$ in *E-system*. Here and further it's necessary to attention to combination in letters markings of *thin* and *bold* types. Present point is essential, so far as $\vec{\boldsymbol{v}}_d \wedge \vec{\boldsymbol{a}}_d$ are velocity and acceleration of *full* deformation, which components comes in matrix of corresponding tensors $(\dot{\mathbf{S}} \vee \dot{\mathbf{A}}) \wedge (\ddot{\mathbf{S}} \vee \ddot{\mathbf{A}})$ of Eulerian description of the *action* functions in dynamics of continuous mediums.

Tensors $\overset{\iota}{\mathbf{A}}$ defines directly components of rotors $\overset{\iota}{\vec{\mathbf{\Omega}}}$ of vector fields $\overset{\iota}{\vec{w}}(\boldsymbol{t},\vec{\mathbf{x}})$ in *E-system*.

Following of stated reasons under limitation by class of functions, wich admits determination, and lowering simples but sufficiently cumbersome transformations of a vector-tensor algebra and analysis, we arrive to following, more total in comparison with formalism, stated in work [3], and closed by the phenomenological enclosures, system of fundamental equations of continuous mediums dynamics (see also clarifications of the notations in [1-3])

$$\frac{d\ln\rho}{dt} = -\vec{\nabla}_{\vec{\mathbf{x}}}\cdot\overset{\bullet}{\vec{w}}; \tag{1}$$

$$\rho\frac{d\vec{v}}{dt} = \rho\vec{F} - \vec{\nabla}_{\vec{\mathbf{x}}}p + 2\vec{\nabla}_{\vec{\mathbf{x}}}\cdot(\overset{\iota}{G}\overset{\iota}{\mathbf{S}}_d) + (\overline{\overset{\iota}{R}\overset{\iota}{\vec{\boldsymbol{l}}}}\cdot\vec{\nabla}_{\vec{\mathbf{x}}})\overset{\iota}{\vec{\mathbf{\Omega}}}; \tag{2}$$

$$\rho\frac{d\varepsilon}{dt} = \mathbf{P}\cdot\cdot\dot{\mathbf{S}} + \vec{\mathbf{M}}\cdot(\vec{\nabla}_{\vec{\mathbf{x}}}\times\vec{v}) + \rho\, q_{cd} + \rho\, q_r; \tag{3}$$

$$\vec{\mathbf{M}} = (\vec{\nabla}_{\vec{\mathbf{x}}}\vec{w})^*\times\mathbf{P} + \overset{\iota}{N}\overset{\iota}{\vec{\mathbf{\Omega}}}; \tag{4}$$

B equations (2), (4) realizes summation to dumb index ι. $\vec{\mathbf{M}}$ is momentum of inner force couples.

In the capacity of additional information, explaining expressions on the right-hand side equations (1) – (4), we point out

$$\mathbf{P} = -p\mathbf{I} + \mathbf{P}_d,\ \mathbf{P}_d = 2\overset{\iota}{G}\overset{\iota}{\mathbf{S}}_d;\ \overset{\iota}{\mathbf{S}}_d = \overset{\iota}{\mathbf{S}} - \frac{1}{3}\overset{\iota}{I}_{s.1}\mathbf{I},\ \ \overset{\iota}{\mathbf{S}} = \left(\overset{\iota}{S}_{ij}\right) = \left(\overset{\iota}{S}_{ji}\right), i\wedge j = \overline{1,3}; \tag{5a}$$

$$\overset{\iota}{\vec{\mathbf{\Omega}}} = (\vec{\nabla}_{\vec{\mathbf{x}}}\times\overset{\iota}{\vec{w}})\vee\overset{\iota}{\vec{\mathbf{\Omega}}} = 2i_k\overset{\iota}{A}{}^*_{lm}\ (k=1,2,3; k\to l\to m \text{- circular permutation}); \tag{5b}$$

$$\overset{\iota}{\vec{\boldsymbol{l}}} = \overset{\iota}{\vec{\mathbf{\Omega}}}/\overline{\overset{\iota}{\Omega}};\quad \overset{\iota}{\vec{w}} = \overset{\iota}{\vec{w}}_f + \overset{\iota}{\vec{w}}_q;\quad \left(\overset{\iota}{A}{}^*_{lm}\right) = \left(\overset{\iota}{A}_{ml}\right)\ni\overset{\iota}{\mathbf{A}}{}^* = -\overset{\iota}{\mathbf{A}}; \tag{5c}$$

$$(\vec{\nabla}_{\vec{\mathbf{x}}}\vec{u})^*\times\mathbf{P} = \frac{\partial\vec{x}}{\partial\mathbf{x}_i}\times(\vec{n}\cdot\mathbf{P})_i,\ \text{(summation to index } i\text{)},\ edo: \frac{\partial\vec{x}}{\partial\mathbf{x}_i} = \frac{\partial\vec{u}}{\partial\mathbf{x}_i}, \vec{u} = \vec{w}; \tag{5d}$$

$$\vec{v} = \vec{v}_f + \vec{v}_q,\quad \vec{v}_q = \mathcal{A}\vec{\nabla}_{\vec{\mathbf{x}}}T,\ \frac{d\vec{v}_q}{dt} = \frac{\partial\vec{v}_q}{\partial t} + \left(\vec{v}\cdot\vec{\nabla}_{\vec{x}}\right)\vec{v}_q;\quad q_{cd} = \vec{\nabla}_{\vec{\mathbf{x}}}\cdot\left(\lambda\vec{\nabla}_{\vec{\mathbf{x}}}T\right) \tag{5e}$$

$$\left(\partial\overset{\iota}{w}_i/\partial\mathbf{x}_j\right)\wedge\overset{\iota}{\mathbf{S}}\wedge\overset{\iota}{\mathbf{A}} \Leftarrow (\overset{\iota}{\vec{w}}_0, \partial\vec{v}/\partial\boldsymbol{t}), (\iota = \varnothing,\bullet),\ i\wedge j = \overline{1,3}; \tag{5f}$$

$$p\wedge T\wedge\overset{\iota}{G}\wedge\overset{\iota}{R}\wedge\overset{\iota}{N}\wedge\lambda\wedge\mathcal{A}\Leftarrow(\rho,\varepsilon). \tag{5g}$$

Second radiation energy entering is q_r defined, as is known, by temperature and physical properties of radiation source. Modules of shear $\overset{\iota}{G}$, torsion $\overset{\iota}{R}$ and torsion momentum $\overset{\iota}{N}_{\mathrm{t}}$ are subjects to partial or total its verification. Symmetry of strain tensor **P** and tensors of deformation, its velocity and acceleration $\overset{\iota}{\mathbf{S}}$, in accordance to equalities (5a), follows from restriction of linear statement for expression coupling between tensors $\mathbf{P} \wedge \overset{\iota}{\mathbf{S}}$ and satisfaction of the Cauchy conditions at small distortions of volume and form of medium marked particle.

We cite the other not mentioned previously notations: *p, T*– pressure and absolute temperature; $\vec{v}_f \wedge \vec{v}_q$ – components of velocity $\vec{v}$ from joint action of a force and heat fields; $\lambda \wedge \mathsf{A}$ – coefficients of thermal conductivity and velocity of <u>thermodeformation</u>; ι = Ø, ●, ●● everywhere excepting (5f); $\vec{n}$ – is unit external normal to the particle boundary; *edo* – Lat. “so as”; i_k– orts of Cartesian coordinate system; pointer ⇐ indicates on one-valued dependence of functions at the left from functions on the right; lower indexes *f, q,* 0 – is reference functions to force or heat fields and initial conditions; signs $\wedge, \vee, \ni$– logical *«and», «or», «so that».*

For components of tensors $\overset{\iota}{\mathbf{S}} \wedge \overset{\iota}{\mathbf{A}}$ (ι = Ø, ●), which are defined, as understood, in *E-system*, in accordance to [1] we have

$$\overset{\iota}{S}_{ij} \vee \overset{\iota}{A}_{ij} = \frac{1}{2}\left\{\left(\frac{\partial \overset{\iota}{w}_{0i}}{\partial \mathbf{x}_j} + \overset{\iota}{Y}_j(\boldsymbol{a}_{pi})\right) + \vee - \left(\frac{\partial \overset{\iota}{w}_{0j}}{\partial \mathbf{x}_i} + \overset{\iota}{Y}_i(\boldsymbol{a}_{pj})\right)\right\}, \tag{6}$$

Where are used operators $Y_{j\vee i} = \int_0^t \left(\int_0^{\leftarrow t} \frac{\partial_\cup}{\partial \mathbf{x}_{j\vee i}} d\tau\right) d\tau$, $\dot{Y}_{j\vee i} = \int_0^t \frac{\partial_\cup}{\partial \mathbf{x}_{j\vee i}} d\tau$.

Unknown functions in equations (1) – (4) are $\vec{v}_f, \rho, \varepsilon, \vec{\mathbf{M}}$.

Specific character of formalism (1) – (4), which distinguish it’s from the mathematical model in work [3], and, under residual symmetrical tensors $\mathbf{P} \wedge \overset{\iota}{\mathbf{S}}$, justify term *quasi-symmetrical* thermomechanics, is contained in following.

- Functions, called as $\overset{\iota}{\vec{w}}$, are relative exceptionally to factors of full deformation.
- Last member to the right in equation (2) is exterior forces by volume effect, which are emerging at torsions and are acting to direction of the $\overset{\iota}{\vec{\mathbf{\Omega}}}$ with its ort $\overset{\iota}{\vec{l}}$ and modules $\overset{\iota}{R}$ of longitudinal deformations from torsion. Given term introduced phenomenologicaly and take into account the initiation of further stresses in consequence of longitudinal distortions of elementary length of matter ‘‘threads’’ / «material» trajectories which accompany effects of rotations.
- Second term in equation (3) on the right represents power of the momentum $\vec{\mathbf{M}}$ of inner force couple, as effect of direct influence on rate of change of internal energy ε.

- Equation (4) for momentum $\vec{\mathbf{M}}$ includes two items: moment of *unbalance* from reduced to unit area the *outside surface* forces ($\vec{n}\cdot\mathbf{P}$), which appears at torsional deformations, and properly torsion momentum of the *outside volume force couples* with modules $\overset{t}{N}$ of torsion moment. Here $\vec{n}$ – is ort of the external normal to surface of the medium particle (see too (5d)).

- Action of heat factors on *all* parameter and *reflective* functions of the field *p*, *T*, i.e. on functions of representation of the substances ρ, ε и $\vec{v}$, change, is defined of theirs (factors) direct dependence from internal energy ε (see 5g), but on two other (except for ε) fundamental substances $\rho \wedge \vec{v}_f$ implicitly by common integrity of the formalism (1) – (6).

At practical calculations important meaning have development of the correct an algorithm of the computing procedure for substantial/individual derivative $\frac{d\mathbf{f}}{dt} = \frac{\partial_t \mathbf{f}}{\partial t} + (\vec{v}\cdot\vec{\nabla}_{\vec{x}})\mathbf{f} = \frac{\partial_t}{\partial t}\mathbf{f} + \frac{\partial_v}{\partial t}\mathbf{f}\; edo\, \partial x_k = v_k \partial t \ni (\vec{v}\cdot\vec{\nabla}_{\vec{x}}) = \frac{\partial_v}{\partial t}$, (7)

Appearing into left parts of the equations (1) – (3).

In equalities (7): notation $\vec{\nabla}_{\vec{x}}$ – is Hamilton's operator in L-system $(t, \vec{x}(t))$, $\mathbf{f}(t, \vec{x}(t))$ – is some function of time and its bearers, i.e. elementary macroparticles of medium, time-varying in mentioned system of «tracking» with local (first item in (7)) and convective (second item in (7)) accelerations. On the basis of stated we comes to next limiting equality

$$\frac{d\mathbf{f}}{dt} = \lim_{\Delta t\to 0}\left(\frac{\mathbf{f}(t+\Delta t, \vec{x}(\tau_t)) - \mathbf{f}(t-\Delta t, \vec{x}(\tau_t))}{2\Delta t} + \frac{\mathbf{f}(\tau_v, \vec{x}(t)+\vec{v}(\tau_v)\Delta t) - \mathbf{f}(\tau_v, \vec{x}(t)-\vec{v}(\tau_v)\Delta t)}{2\Delta t}\right), \quad (8)$$

$$\tau_l = (\alpha_l(t-\Delta t) + \beta_l(t+\Delta t)) \in [t-\Delta t, t+\Delta t]\,,\; \alpha_l + \beta_l = 1\,,\; l = t \vee v\,,$$

Representing itself nonlinear *congregatory* substantial derivative from **f** to t at explicit expression of the given function only from the same absolute argument, namely from time.

As is evident, in term of (8) scheme with central differences is used. Parameters $\tau_t \wedge \tau_v$ – are «weighted mean» values time, from indicated interval in the limiting definitions of the partial derivatives to time for explicit dependence of the **f** from *t* and for transfer along its trajectory respectively. These parameters takes into account degree of irregularity of function **f** change on the pertaining to time quantum 2Δ*t*. Fur the $\alpha_l \wedge \beta_l$ – are weight coefficients of identification of this circumstance. Indicated coefficients establishes by rational prescribed algorithm.

For scalar functions **f** limiting passages (8) are realized directly, and for vector functions (see (1) – (3))–to each their component. Obviously also, that $\partial_t \mathbf{f} / \partial t = 0$ at steady state, and by motion of medium particles with constant meaning of field functions along rectilinear coordinate of the inertial reference sys $d\mathbf{f}/dt \equiv 0$. Besides, it is not difficult make sure into immanent convergence of the partial derivatives from **f** to $t \wedge \boldsymbol{t}$ in $L \wedge E$-reference systems for unstationary of motion conditions, as in the first case is considered motion of individual *marked* particle, and in the second case is observed the passing of focus of it particle identities to time across point $\vec{\mathbf{x}}$ of Eulerian space.

We still note, that on attainment of a steady state right parts of equations (1) – (3), i.e. functions of action, in general are non-uniform to space, but are stationary, i.e. independent from ***t***, and left parts, i.e. velocities of change of field substances, are functions only convective transfer of indicated monad without autonomous dependence from t, i. e. at zero meaning of first limit in expression (8).

CONCLUSION

It`s shown, that dynamics of continuous mediums in common case mast be regarded to *quasi-symmetrical* statement. This conclusion is based on the condition, that torsions of medium particles relatively of their focus realizes to regularities of compressable deformable and heat-conducting mediums.

It`s competed transfer at proper modification of phenomenological model of the continuous medium termomechanics.

It`s defined as function absolute argument, namely time and composed nonlinear substantial derivative for fundamental monad of field in Lagrangian reference system.

REFERENCES

1. Morgunov G. M. Deduction of action factors in Neoformalism of dynamics of continuous media // Materials of the VIII ISPC “Academ. Science-problems and achievements”, v.1, 15-16 Feb. 2015. – North Charleston, USA, 2016, pp. 142-146, ISBN: 978-1530131884.

2. Morgunov G. M. Truth episteme in continuous mediums dynamics // Materials of the VI ISPC “Topical cureas of fundamental and applied research’’, v.1, 22 – 23 June. 2015. - North Charleston, USA, pp. 144-148. – ISBN 978-15147117370

3. Morgunov G. M. Neoformalism approach to the dynamics of continuous media / World intellectual property organization. Publ. date 05.02.2015: 44 p. [in Russian]. https://patentscope.wipo.int/search/en/detail.jst?docId=W02015016736